

Maximal Sublattices of Finite Distributive Lattices: A Problem from the 1984 Banff Conference on Graphs and Order

Jonathan David Farley
Department of Mathematics and Computer Science
University of the West Indies Jamaica

February 1, 2007

Let L be a finite distributive lattice. Let $\text{Sub}_0(L)$ be the lattice $\{S \mid S \text{ is a sublattice of } L\} \cup \{\emptyset\}$ and let $\downarrow_*[\text{Sub}_0(L)]$ be the length of the shortest maximal chain in $\text{Sub}_0(L)$. It is proved that, if K and L are non-trivial finite distributive lattices, then

$$\downarrow_*[\text{Sub}_0(K \times L)] = \downarrow_*[\text{Sub}_0(K)] + \downarrow_*[\text{Sub}_0(L)].$$

A conjecture from the 1984 Banff Conference on Graphs and Order is thus proved.