ABSTRACT

"Graphs are used to model a wide variety of phenomena, ranging from ecological systems to river flow to the WWW network. A primary goal of algebraic graph theory is to understand the relationship between the combinatorial structure of a graph and the algebraic structure of associated matrices. For example, the eigenvalues of the Laplacian matrix of a graph are closely related to the interconnectedness of the graph. As the Laplacian can be viewed as a discrete version of the continuous Laplacian, in essence one is using discrete harmonic analysis to try to "hear" the combinatorial properties of a graph."

We consider only simple graphs. Given two graphs G with vertices $1, \cdots, n$ and H the corona $G \circ H$ is defined as the graph obtained by taking n copies of H and for each i inserting edges between the ith vertex of G and each vertex of the ith copy of H. It is well known that a graph G is bipartite if and only if the negative of each eigenvalue of G is also an eigenvalue of G. We ask a similar question of characterizing all graphs for which the reciprocal of each eigenvalue is also an eigenvalue with equal multiplicity (graph with property (R)). We characterize all such trees, called as corona trees. We supply a family of bipartite graphs with property (R). Suitable examples are given to show that a graph with property (R) need not be a corona of two graphs and need not be bipartite. For a connected graph G and any r-regular graph H we provide complete information about the spectrum of $G \circ H$ using the spectrum of G and spectrum of H. Complete information about the Laplacian spectrum of $G \circ H$ is also provided even when H is not regular. As an application we show how to construct infinitely many pairs of nonisomorphic graphs with the same spectrum and the same Laplacian spectrum.