Objectives for the First, Second and Third Midterm Exams, and in addition

State the Definitions and Use Them in Proofs.

● Series
  – Series, partial sum
  – Convergent series, divergent series
  – Absolutely convergent series, conditionally convergent series.

● Limits of Functions, Continuity
  – Cluster point of a set $S \subset \mathbb{R}$.
  – Isolated point of a set.
  – Limit of a function at a cluster point of its domain.
  – Restriction of a function to a subset of its domain.
  – One-sided limit of a function at a point.
  – Continuity of a function at a point of its domain.
  – Continuous function on a set.

State the Theorems and Use Them in Proofs.

● Series
  – A series converges if and only if every one of its tail series converges.
  – Cauchy convergence criterion for series.
  – Necessary condition for convergence of a series
    (also known as the divergence test).
  – Convergence criterion (and sum when convergent) for geometric series.
  – Divergence of the harmonic series.
  – Linear operations on convergent series.
  – An absolutely convergent series is convergent.
  – Comparison test.
  – Necessary and sufficient condition for convergence of the series $\zeta(p) = \sum_{n=1}^{\infty} n^{-p}$.
  – Ratio test for series.
  – Triangle inequality for series.
  – Limit comparison test.

● Limits of Functions, Continuity
  – Necessary and sufficient condition for a point to be a cluster point of a set.
  – Uniqueness of the limit of a function at a point.
  – A function has a limit at a point if and only if it has a sequential limit at that point.
  – Squeeze lemma for limits of functions.
  – Limits of sums, products and quotients.
  – Continuity is equivalent to sequential continuity.
  – The composition of continuous functions is continuous.
  – Examples of extremely discontinuous functions.
  – A continuous function $f : [a, b] \to \mathbb{R}$ is bounded.
  – Maximum-minimum theorem for continuous functions.
  – Bolzano’s intermediate value theorem for continuous functions.