Procedure to Determine Convergence or Divergence of a Series

Math 166 Fall 2005

Case 1. $\sum_{n=1}^{\infty} a_n$, $a_n > 0$ (positive terms)

Test 1. n-th term test (NTT): If $\lim_{n\to\infty} a_n \neq 0$ then series diverge

Note: if $\lim_{n\to\infty} a_n = 0$ then series has a chance to converge, must do more tests.

Test 2. If an involves: $n!$, r^n, n^n : try Ratio Test (CRT)

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

converges if $\rho < 1$
diverges if $\rho > 1$
inconclusive if $\rho = 1$

Note: if $\rho = 1$ when using Ratio Test, you need to do more tests.

Test 3. If a_n involves polynomials in n (n^2, n^3, n^5, etc.) then using Limit Comparison Test (LCT)

LCT: Given Σa_n, guess if Σa_n converges or diverges. Then pick Σb_n so that Σb_n matches your guess for Σa_n. Compute

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}.$$

If $0 < L < \infty$ then Σa_n and Σb_n converge or diverge together.
If $L = 0$ and Σb_n converges, then Σa_n converges.
If $L = \infty$ the test is inconclusive & you must either pick a different Σb_n or a different test.

Test 4. If the above tests don't work, try: Ordinary Comparison Test
Integral Test
Bounded Sum Test.

OCT: $a_n < b_n$ and $\Sigma b_n < \infty \Rightarrow \Sigma a_n < \infty$.

IT: $\Sigma a_n < \infty \iff \int_1^{\infty} a_n \, dn < \infty$

BST: $S_n = \sum_{k=1}^{n} a_n \leq B$ for all n means $S = \lim_{n\to\infty} S_n \leq B + \sum_{n=1}^{\infty} a_n < \infty$

Case 2. Alternating series $\sum_{n=1}^{\infty} u_n$.

Test 2. Try Alternating Series Test:
If $\lim_{n\to\infty} |u_n| = 0$ and $|u_n| > |u_{n+1}|$ then $\sum u_n$ converges.

Test 1. If $\lim_{n\to\infty} |u_n| \neq 0$ series diverges.

Test 3. To test for absolute convergence, check $\sum_{n=1}^{\infty} |u_n|$ for convergence. Here $|u_n| = a_n > 0$, and thus $\sum_{n=1}^{\infty} |u_n|$ is a positive series, and all the tests for Case 1 series apply. If you get divergence of $\sum_{n=1}^{\infty} |u_n|$ but convergence of $\sum_{n=1}^{\infty} u_n$, then you have conditional convergence.