

Name _____

Instructor _____

Math 165 Midterm, Spring 2007

For full credit, all work must be shown. No calculators may be used on this portion. You will have 40 minutes to complete problems 1.-10.

For 1.-4., evaluate the limit.

1. (6 pts) $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

2. (6 pts) $\lim_{\theta \rightarrow \pi} \frac{\sin \theta}{\theta}$

3. (6 pts) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1}$

4. (6 pts) $\lim_{x \rightarrow \infty} \sqrt{\frac{4x^2 - 7x}{1 + x^2}}$

5. (6 pts) Evaluate the following limit by recognizing it as a derivative:

$$\lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

For 6.-9., find $\frac{dy}{dx}$.

6. (6 pts) $y = x^2 + \sin^2(x^2 + 1)$

7. (6 pts) $y = \pi$

8. (6 pts) $y = \frac{x + 1}{x - 1}$

9. (6 pts) $x^2y^2 + 4xy = 0$

10. (6 pts) Find $\frac{d^2y}{dx^2}$, where $y = x \sin x + \cos x$.

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For full credit, all work must be shown and *exact* answers given.

Calculators MAY be used on this portion.

11. (a) (6 pts) Let $g(t) = \frac{\tan(5t)}{8t}$ for $t \neq 0$. Can we define $g(0)$ so that g is continuous everywhere? If so, define it; if not, explain why.

- (b) (6 pts) Let $h(t) = \frac{\cos(2t)}{7t}$ for $t \neq 0$. Can we define $h(0)$ so that h is continuous everywhere? If so, define it; if not, explain why.

12. (12 pts) Let $f(x) = \frac{x^2}{2} + \tan(\pi x) + 2\pi$. Find the equation for the line tangent to the curve at the point $x = 0$.

13. (12 pts) A large, very buoyant weather balloon is released at a point 1 mile from an observer (who is on the ground). If the balloon goes straight up at a rate of 52.8 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 2 miles high? *NB*: there are 5280 feet in a mile.

14. (a) (8 pts) Use differentials (or a linear approximation) to find a rational approximation to $\sqrt{50}$. (No credit for an answer without supporting work!)

(b) (4 pts) Explain why the estimate in part (a) will be an *overestimate*. (Hint: draw a picture!)