Math 265 **Name & Section:** ____________________

Final Exam 2007/04/30 **Instructor:** ____________________

Answer each question completely. Show all work. No credit is allowed for mere answers with no work shown. Show the steps of calculations. State the reasons that justify conclusions.

1. (12 points) Show that the triangle with vertices $P = (4, 1, 3)$, $Q = (2, 0, 5)$ and $R = (4, 2, 8)$ is a *right triangle*. Which vertex has the right angle?

2. (12 points) Find the point(s) on the upper half of the ellipsoid $2x^2 + y^2 + 3z^2 = 1$ where the tangent plane is parallel to the plane $4x - 4y + 6z = 15$.

3. (14 points) Find all critical points of $f(x, y) = \frac{1}{x} + xy + \frac{1}{y}$. Indicate whether each such point gives a local maximum or a local minimum, or whether it is a saddle point.

4. (12 points) Find the volume of the solid in the first octant bounded by the coordinate planes and the planes $2x + y - 4 = 0$ and $8x - 4z = 0$.

5. Write the iterated integral (but do not evaluate it)

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$$

as

(a) (6 points) an iterated integral in cylindrical coordinates;

(b) (6 points) an iterated integral in spherical coordinates.

6. Let $\mathbf{F}(x, y) = (ye^{xy} + y)\,\mathbf{i} + (xe^{xy} + x)\,\mathbf{j}$.

(a) (7 points) Determine whether $\mathbf{F}$ is conservative. If so, find f so that $\mathbf{F} = \nabla f$. If not, state that $\mathbf{F}$ is not conservative.

(b) (7 points) Let C be the perimeter of the triangle with vertices $(1, 2)$, $(3, 7)$, $(-2, -1)$, oriented counterclockwise. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

7. (12 points) Evaluate $\displaystyle\oint_C \ln(1+y)\,dx + \left(3 + \frac{1}{1+y}\right) x\,dy$ where C is the boundary of the triangle with vertices $(0,0)$, $(4,0)$ and $(0,4)$, oriented counterclockwise.

8. (12 points) Evaluate $\iint_{\partial Q} \mathbf{F} \cdot \mathbf{n}\, dS$. The vector field $\mathbf{F} = (y-x)\,\mathbf{i} + (z-y)\,\mathbf{j} + (y-x)\,\mathbf{k}$. The solid Q is the cube bounded by the planes $x = \pm 1$, $y = \pm 1$ and $z = \pm 1$.