

Iowa State University
Math 166 - Spring 2007

FINAL EXAM
MAY 3, 2007

Name: _____

Instructor: _____ TA: _____ Section: _____

Instructions:

- You have 120 minutes to complete the exam.
- Books, class notes or computers cannot be used during the exam.
- You need to show your work to get full credit. Correct answers without explanation or with incorrect reasoning will not get full credit.
- State the reasons that justify your conclusions.
- Mention theorems from the book and in-class results when appropriate.
- Use the space below the problem statement to derive your answer.
- Clearly mark your final answer.

1. Determine if the following series converges absolutely, converges conditionally, or diverges. State your reasons.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$$

2. Determine if the following series converges absolutely, converges conditionally, or diverges. State your reasons.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$$

3. Find the convergence set of the following power series. Which test(s) do you use?

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n}$$

4. Find the power series representation for

$$f(x) = \int_0^x e^{-t^2} dt.$$

Hint: Start with the Taylor series $e^u = \sum_{n=0}^{\infty} \frac{1}{n!} u^n$.

5. (a) Find the Maclaurin polynomial of order 3 for the function $f(x) = e^{-x}$.

(b) Estimate the error you make if you use this polynomial to approximate $f(x)$ at $x = 0.1$.

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FINAL EXAM - PART B
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6. Find the area of one leaf of

$$r = 5 \cos 2\theta.$$

7. Evaluate the following improper integral or show that it diverges.

$$\int_0^2 \frac{x}{\sqrt{4-x^2}} dx$$

8. Find the antiderivative

$$\int t^5 \ln(t^7) dt.$$

9. Find the length of the curve generated by the parametric equations

$$x = -\cos t, \quad y = \pi + \sin t, \quad 0 \leq t \leq \pi.$$

10. Consider a tank in the shape of a right circular cone with base resting on the ground. The radius at the base is 4 ft., the height of the cone is 10 ft. The tank is filled with a liquid of density δ . Find the work done in pumping all the liquid to a height of 5 ft. above the top of the tank.