Chapter 14

FLOWS AT HIGHER REYNOLDS NUMBERS

14.1 Introduction

Most of our work has been done at a Reynolds number $Re = 1$; real fluid flows encountered in aircraft engine design may have Reynolds numbers of 10,000, 100,000 or more. We discuss in this chapter some of the changes expected in the behavior of the flow and in the form of the sensitivities as Re increases. Because higher Reynolds number flows are inherently more difficult to compute, we discuss a continuation method approach for producing a suitable starting point for the Newton iteration.

14.2 "Local" Influences Become Global

Low Reynolds number flow could be characterized by honey moving through a tube, while an example of high Reynolds number flow would be air driven at high speed down a tunnel. Experience tells us that the high Reynolds number flow is inherently more unpredictable and unsteady. We are interested, though, in the behavior of the two kinds of flows when an obstacle is encountered. We have already seen that, for $Re = 1$, the velocity sensitivities are relatively large near the bump, but die out very quickly downstream. When we plotted the sensitivities, we saw a "whirlpool" of influence right around the bump, and little else. We interpreted this to mean that the bump caused almost purely a local disturbance.

It is natural to suppose, again from physical experience and intuition, that as the Reynolds number increases, the influence of the bump will extend to a greater proportion of the flow region behind the bump, while decreasing somewhat in front of the bump. In fact, at higher Reynolds numbers, the influence of *all* the parameters and boundary conditions begins to affect the solution throughout the region.

This is good news for our optimization efforts; it means that more of the flow will be affected by the bump, and we will have a greater choice on where we place our profile line, or, indeed, on what quantities we measure there.

On the other hand, it is going to make our individual computations of particular flow solutions much harder; as Re increases, the nonlinear terms begin to dominate and our problem, which we symbolize abstractly as $F^h(X) = 0$, becomes much harder to solve.

As Re increases, $F^h(X) = 0$ becomes not only a harder problem, but also a *bigger* problem, because the mesh parameter h must be decreased. Cutting h in half quadruples the number of unknowns, and our banded system matrix increases in size by roughly a factor of 8. Such growth factors cannot be long sustained, and severely limit the Reynolds numbers for which we can compute an approximate flow solution. But if the mesh is not sufficiently refined for a given Re, the Newton iteration may converge extremely slowly, or, more likely, will almost immediately diverge.

We now exhibit a set of plots of the α-velocity sensitivity field for Re=1, 10, 100, 500 and 1,000. These plots confirm the prediction that the influence of the bump begins to extend downstream; it is almost as though the "whirlpool" that showed up above the bump, for

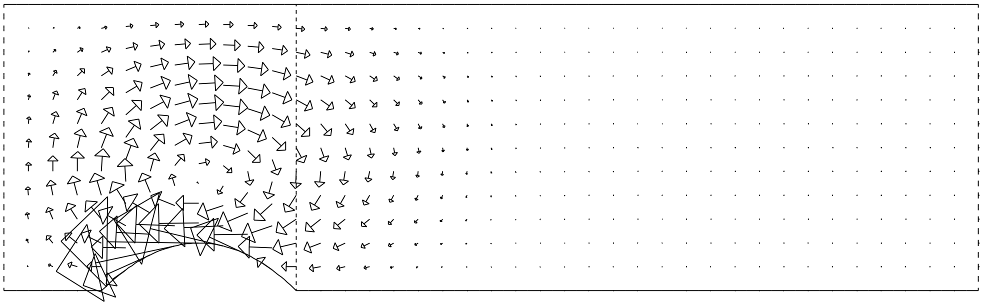

Figure 14.1: Velocity sensitivities for $Re = 1$.

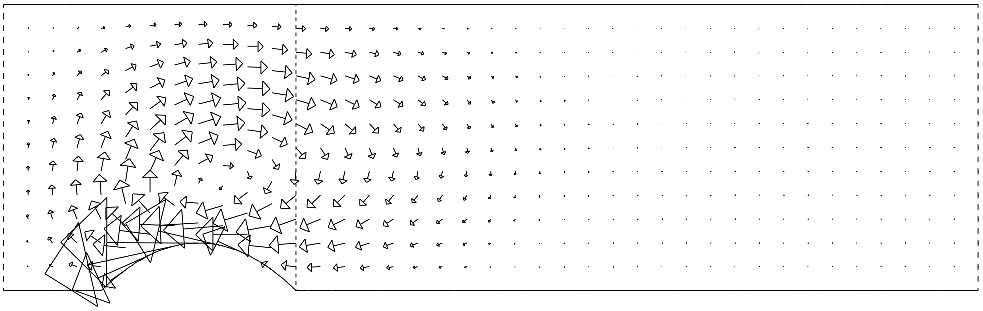

Figure 14.2: Velocity sensitivities for $Re = 10$.

$Re = 1$, is blown to the right by an incoming wind.

A comparison of the sensitivity plots for $Re = 1$ and 1000 suggests that, at $Re = 1$, the disruptions that the bump causes are strictly confined to the region above the bump, where more flow has to pass through a narrowed channel; once past the bump, the flow quickly restores itself to its original pattern. For $Re = 1000$, however, the flow deflected upward does not come back down. The large arrows in the wake of the bump indicate that the bump is causing the flow just behind it to die off.

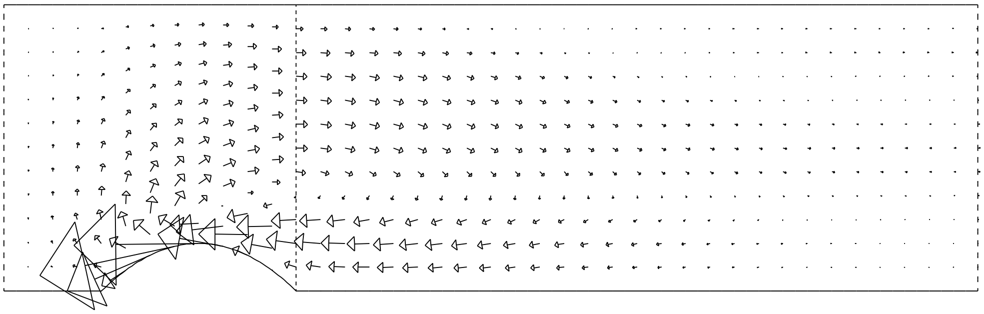

Figure 14.3: Velocity sensitivities for $Re = 100$.

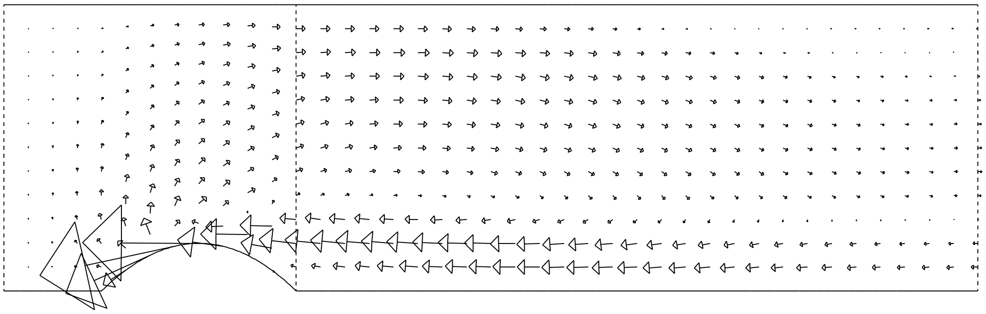

Figure 14.4: Velocity sensitivities for $Re = 500$.

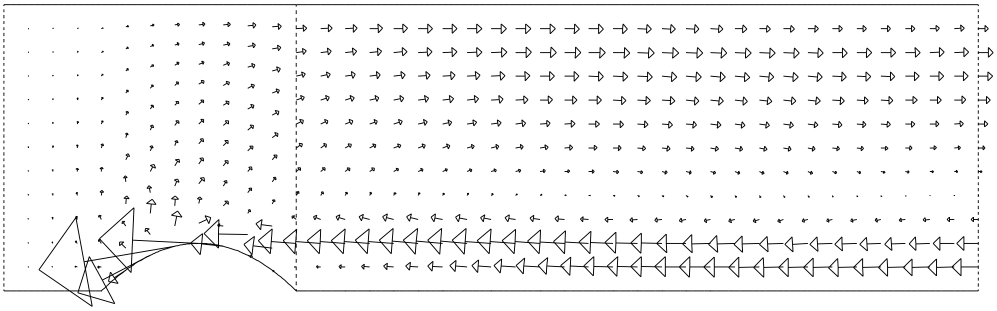

Figure 14.5: Velocity sensitivities for $Re = 1000$.

14.3 Getting a Starting Point

At low Reynolds numbers, we started our first Newton process with a zero guess, and thereafter simply used the previous solution for the next Newton process. At higher Reynolds numbers, the nonlinear terms are more significant. This means that Newton's method has to work harder, and requires a better starting point. It's easy to detect when the starting point isn't good enough, because the Newton iteration will begin to fail.

If a better starting point is needed, one option is to compute a solution using a simple form of *continuation* [23]. That is, to get the solution (u, v, p) for the parameters $(\lambda_0, \alpha_0, Re_0)$, we first compute the solution for $(\lambda_0, \alpha_0, 1)$. For example, if Re_0 is "close" to 1, and we have a flow solution computed at $Re = 1$, then we can compute a good initial estimate of the solution at Re_0 by using an Euler approximation:

$$u(\lambda_0, \alpha_0, Re_0) \approx u(\lambda_0, \alpha_0, 1) + \frac{\partial u}{*} \partial Re (Re_0 - 1). \tag{14.1}$$

Similar formulas are required to approximate the new values of v and p. Notice that, once again, we need the sensitivities of the solution variables, this time with respect to Re. Of course, we can estimate the sensitivities by finite differences, or try to economize by using discretized sensitivities.

If Re_0 is not close to 1, then the Euler approximation might not produce a good enough starting point for the Newton iteration. Then a possible rememdy would be to advance to Re_0 in a sequence of several steps. At each step, we use an Euler approximation based at the previous step as the starting point for our Newton process.

Once we have gotten a single solution (u, v, p) at our parameter values $(\lambda_0, \alpha_0, Re_0)$, we may choose to reuse that solution as the starting point for the next Newton process. However, at high Reynolds numbers, the solution is much more sensitive to the parameter values, and we may find this approximation unsatisfactory. In that case, we must return to the continuation method. If the Newton process fails during a continuation step, we always have the option of reducing the stepsize and trying again.

We have used this approach successfully to compute target solutions at Reynolds numbers of 100 and 1,000. However, during the subsequent optimization steps, we have encountered numerous problems with Newton divergence. In some cases, the bump parameter values become strongly negative, or oscillatory; in other cases there is no clear reason for the failure. Frequently, a convergence could be corrected by refining the mesh. However, because of computer memory limitations, it was not possible for us to use a mesh parameter h finer than 0.0625, which meant we frequently were unable to solve problems at $Re = 1000$.