SENSITIVITY ANALYSES AND COMPUTATIONAL SHAPE OPTIMIZATION FOR INCOMPRESSIBLE FLOWS

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ABSTRACT

We consider the optimization of a cost functional defined for a fluid flowing through a channel. Parameters control the shape of an obstruction in the flow, and the strength of the inflow. The problem is discretized using finite elements. Optimization algorithms are considered which use either finite differences or sensitivities to estimate the gradient of the cost functional. Problems of scaling, local minimization, and cost functional regularization are considered. Methods of improving the efficiency of the algorithm are proposed.

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Finally, I wish to thank my mother and father, who always believed in the value of learning, who were patiently supportive through my prolonged and discursive education, and who are possibly more excited about my degree than if they had gotten one themselves.

DEDICATION

My sister Barbara was born when I was 11, and she spent most of her early years riding on my shoulders. When our family moved across the country in a trailer, it was because of her that I got to sleep in the lowest, biggest bed (and it was over me that she had to crawl when she managed to fall out of that bed). When she got too big to carry on my shoulders, I used to swing her around, first by her arms, and then by her legs, and I only remember dropping her once. When I went to high school, she used to wait for me by the front door in the afternoon. When I went off to college, I left her on her own, but she turned out pretty well anyway. Now that she works at Hershey, she's taking care of me with regular shipments of chocolate.

In honor of our long happy years together, this thesis is dedicated to my dear sister Barbara.

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VITA

The author's improbable career may bring hope to those still struggling to find their proper place in life.

He went to MIT with a vague interest in Mathematics, only to be blighted by a course in number theory. An attempt at a physics major came to an end in the lab, when he managed to make a vacuum tube with two anodes. He started a computer course, but couldn't get the hang of it. Almost every term, though, he took courses in German and in writing. It was only by the purest luck that he was able to graduate, as the first recipient of a degree in the newly blessed field of "Science and Humanities".

Graduated, jobless, planless, he decided to take some summer classes at the University of Pittsburgh. He enjoyed his course in sophomore calculus so much that, at the end, he practically begged his instructor, "Is there some way I could do more of this?" The instructor arranged for him to meet Professor Jim Fink, who got him provisional admission, conditional on surviving a term of catchup work. That first term involved back-to-back classes from 9 AM to 3 PM, with no lunch break, but was the first time he had enjoyed school in years. He was so excited he wrote to two old MIT classmates who were currently returning cars for Avis, and they both also enrolled in graduate school, also in Mathematics.

After two years of abstract mathematics, including topology, set theory and logic, he felt unprepared to take the preliminary exams and simply didn't show up, forfeiting all further

departmental support. It was only by a miracle that he found himself saved, working on a one year grant with Professor Andras Szeri of the Mechanical Engineering Department, who required him to learn FORTRAN, the use of a timesharing computer, and numerical analysis.

The course in numerical analysis, taught by Professors Charles Hall and Tom Porsching, is fixed in his mind for all time. He had enjoyed the preliminary material, but expected things were about to go downhill when ordinary differential equations arrived. But then the professor sketched the field of derivatives and said "Suppose we're at point X, where the solution is equal to Y, and we want to estimate the value of the solution at point X + Δ X. What's a simple estimate we can make, using this picture?" This was the moment when he fell in love with numerical analysis.

Gradually, he drifted from graduate work into full time computer support for Hall and Porsching's research group in the Mathematics department. He worked on a variety of fluid codes for the Air Force and EPRI, and a sheet metal forming program for GM. He was lucky enough to work with Professor Werner Rheinboldt in developing a version of his continuation program suitable for submission as an ACM algorithm. He also spent a great deal of time writing educational software for Professor Charles Cullen, from whom he learned a lot about proper computer educational techniques.

He then took a job at the newly opened Pittsburgh Supercomputing Center, as a "mathematical software librarian", which initially meant supporting the IMSL and NAG libraries. Gradually this job came to include installing, testing, and documenting software, porting software to a variety of Cray computers, debugging and optimizing user programs, team-teaching workshops for users, both on site and at a number of universities, and working on an outreach program that brought the joys and headaches of computing to students and teachers in local high schools.

However, after five years, he was ready for something new. Professor Janet Peterson, who had known him at the University of Pittsburgh and was now at Virginia Tech, had urged him several times to return to graduate school and finish his degree. Now she suggested he come down to Virginia Tech and do so. Professor Max Gunzburger also applied some gentle charm, offering to supervise his work, and introducing him to Professors John Burns, Gene Cliff, and Terry Herdman, the likely members of his defense committee.

It seemed like the last chance of a lifetime, and so he quit his job, sold his house, rode down to Blacksburg with six boxes of belongings, and hasn't looked back.