Math 307, Fall 2009 - Quiz 9 Solution

Problem 1.

For the following matrix, find:

1) the eigenvalues (5 points)
2) corresponding eigenspaces (5 points)
3) a specific eigenvector for each eigenvalue. (2 points)
4) choose one eigenvalue and find a basis for its eigenspace (3 points)
5) algebraic and geometric multiplicity of that eigenvalue. (2 points)

$$\begin{pmatrix} 2 & 3 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

Eigenvalues

$$|A-\lambda I| = \begin{vmatrix} (2-\lambda) & 3 & 0 & 0 \\ 0 & (-1-\lambda) & 0 & 0 \\ 0 & 0 & (2-\lambda) & 0 \\ 0 & 0 & -1 & (-1-\lambda) \end{vmatrix} = (2-\lambda)\begin{vmatrix} -1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & -1-\lambda \end{vmatrix} = (2-\lambda)^2(-1-\lambda)^2 = 0.$$

Thus $\lambda_1 = \lambda_2 = 2, \quad \lambda_3 = \lambda_4 = -1.$

Eigenspaces

$\lambda_1 = \lambda_2 = 2.$

$$(A-2I)\mathbf{v} = \begin{pmatrix} 0 & 3 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -3 \end{pmatrix}\mathbf{v} = \mathbf{0},$$

iff

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}\mathbf{v} = \mathbf{0},$$

$$E_2 = \{(t,0,s,-3s) : t,s \in \mathbb{R}\} = Span(\{(1,0,0,0),(0,0,1,-3)\}).$$

A basis of E_2 is $B = \{(1,0,0,0),(0,0,1,-3)\}$, $dim(E_2) = 2$.

$\lambda_3 = \lambda_4 = -1$.

$$(A+I)\mathbf{v} = \begin{pmatrix} 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & \text{-1} & 0 \end{pmatrix} \mathbf{v} = \mathbf{0},$$

iff

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{v} = \mathbf{0},$$

$$E_{-1} = \{(t,-t,0,s) : t,s \in \mathbb{R}\} = Span(\{(1,-1,0,0),(0,0,0,1)\}).$$

A basis of E_{-1} is $B = \{(1,-1,0,0),(0,0,0,1)\}$, $dim(E_{-1}) = 2$.