

### HOMEWORK 3, MATH 607, Fall 2009

#### Problem 3.1.

Let  $G$  be a regular bipartite graph of degree at least 2. Show that  $\kappa(G) \neq 1$ .

#### Problem 3.2.

Let  $G$  be a subgraph of  $K_{n,n}$  with no  $(k+1)$  independent edges. Show that the number of edges in  $G$  is at most  $kn$ .

#### Problem 3.3.

Let  $G$  be an  $r$ -regular graph of odd girth  $g$ . Prove that

$$|V(G)| \geq 1 + r + r(r-1) + \dots + r(r-1)^{\frac{g-3}{2}}.$$

#### Problem 3.4.

Show that the block graph of any connected graph is a tree.

#### Problem 3.5.

Let  $A$  be the adjacency matrix of a graph  $G$  with vertices  $v_1, \dots, v_n$ . Show that the  $i, j$ th entry of  $A^\ell$  is the number of  $v_i - v_j$  walks of length  $\ell$ .

#### Problem 3.6

- a) Find the spectrum of a complete graph  $K_n$ .
- b) The bisection width of a graph  $G$  is defined as

$$b(G) = \min\{|E(S, T)| : S, T \subseteq V(G), ||S| - |T|| \leq 1, S \cup T = V(G), S \cap T = \emptyset\}.$$

If  $G$  is  $r$ -regular with spectrum  $\lambda_0 \geq \lambda_1 \geq \dots \geq \lambda_n$ , find the good bounds for  $b(G)$ .