HOMEWORK 2, MATH 607, Fall 2009

Problem 2.1.
For each choice of integers k, l, m $0 < k \leq l \leq m$ construct a simple graph with $\kappa(G) = k$, $\kappa'(G) = l$, $\delta(G) = m$.

Problem 2.2.
Let G be a connected graph such that for any edge e of G there are cycles C_1 and C_2 with $e = E(C_1) \cap E(C_2)$. Prove that G is 3-edge connected. Using this fact, prove that Petersen graph is 3-edge connected.

Problem 2.3.
Let G be 2-connected graph. For $xy \in E(G)$, prove that $G - xy$ is 2-connected if and only if $x, y \in V(C)$ for some cycle C in $G - xy$.

Problem 2.4.
Let $A = A_1, ..., A_m$ be a collection of subsets of a set Y. A system of distinct representatives for A is the set of distinct elements $a_1, ..., a_m$ such that $a_i \in A_i$ for all $i = 1, \cdots, m$. Prove that A has a system of distinct representatives if and only if $|\cup_{i \in S} A_i| \geq |S|$ for every $S \subseteq \{1, \cdots, m\}$.

Problem 2.5
A permutation matrix is a matrix of zeros and ones such that each row and each column contain exactly one 1. Show that a square matrix A with nonnegative integer entries is a sum of k permulation matrices if and only if the sum of elements in each row and in each column of A is k.

Problem 2.6.
For each integer $k > 1$ construct a k-regular graph with no 1-factor.