HOMEWORK 1, MATH 607, FALL 2009 DUE THURSDAY, SEPT. 3

Solve any four problems.

Problem 1.1
a) Show that if G is a connected graph of order n then for all k, $1 \leq k \leq n$ there is a connected subgraph of G of order k.

b) Prove that either a graph or its complement is connected.

Problem 1.2.
Show that any tree T has at least $\Delta(T)$ leaves.

Problem 1.3.
Let G be a graph with minimal degree at least two. Show that there is a connected graph having the same degree sequence as G.

Problem 1.4
Let G be a connected simple graph which does not have C_3 or P_4 as induced subgraphs. Prove that G is a complete bipartite graphs.

Problem 1.5.
Let T be a tree of even order. Prove that T has exactly one spanning subgraph in which every vertex has odd degree.