

SUMMARY of GEOMETRIC LINEAR TRANSFORMS in \mathbb{R}^2 .

Dilation

$$T(\mathbf{x}) = a\mathbf{x} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \mathbf{x}.$$

Reflexion with respect to x_1 -axis

$$T(\mathbf{x}) = T((x_1, x_2)) = (x_1, -x_2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x}.$$

Reflexion with respect to x_2 -axis

$$T(\mathbf{x}) = T((x_1, x_2)) = (-x_1, x_2) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x}.$$

Projection onto the vector $w = (w_1, w_2)$

$$T(\mathbf{x}) = \text{proj}_{\mathbf{w}} \mathbf{x} = \frac{\mathbf{w}}{|\mathbf{w}|^2} (\mathbf{x} \cdot \mathbf{w}) = \begin{pmatrix} \frac{w_1 w_1}{|\mathbf{w}|^2} & \frac{w_1 w_2}{|\mathbf{w}|^2} \\ \frac{w_2 w_1}{|\mathbf{w}|^2} & \frac{w_2 w_2}{|\mathbf{w}|^2} \end{pmatrix} \mathbf{x}.$$

Reflexion with respect to the vector $w = (w_1, w_2)$

$$T(\mathbf{x}) = 2\text{proj}_{\mathbf{w}} \mathbf{x} - \mathbf{x} = \begin{pmatrix} 2\frac{w_1 w_1}{|\mathbf{w}|^2} - 1 & 2\frac{w_1 w_2}{|\mathbf{w}|^2} \\ 2\frac{w_2 w_1}{|\mathbf{w}|^2} & 2\frac{w_2 w_2}{|\mathbf{w}|^2} - 1 \end{pmatrix} \mathbf{x}.$$

Rotation counterclockwise by the angle ϕ

$$T(\mathbf{x}) = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \mathbf{x}.$$

Horizontal shear with coefficient k $T(\mathbf{x}) = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \mathbf{x}$.

Building a matrix of a transform using standard basis

$$T(\mathbf{x}) = \begin{pmatrix} T(\mathbf{e}_1)T(\mathbf{e}_2)\dots T(\mathbf{e}_n) \end{pmatrix} \mathbf{x}.$$