Problem list for final exam, math 201, Spring 2009.

Problem 1 Prove that the sum of odd number of odd numbers is odd.

Problem 2 Let S be an $n \times m$ grid, $n \leq m$. Find the number of squares in this grid. Formally prove your formula.

Problem 3 Let $x \equiv 3(mod4)$ and $y \equiv 2(mod4)$. Find $x^2+y(mod2)$, $x^2+y(mod4)$, $x^2+y(mod8)$. Justify your answers.

Problem 4 Use mathematical induction to prove that the number of times one need to break an $m \times n$ candy bar into single pieces is $mn-1$.
Bonus points: give a short proof without using induction.

Problem 5 Let $R = \{(A,B) : A \subseteq BorB \subseteq A, A, B \subseteq [n]\}$. Is R an equivalence relation? Let $R' = \{(A,B) : A \subseteq BandB \subseteq A, A, B \subseteq [n]\}$. Is R' an equivalence relation? Provide formal proofs.

Problem 6 Use proof by contrapositive to show that if there are k pencils of 5 different colors in a box, there are at least $k/5$ pencils of the same color in that box.

Problem 7 Consider $\mathbb{Z}_n$. Let $f_n : \mathbb{Z}_n \to \mathbb{Z}_n$ given by $f_n([x]) = [2x+2]$. Is f_n a bijection for any n? Prove or disprove.

Problem 8 Prove that $lim_{x \to 0} 2^{2^{2^x}} = 4$.

Problem 9 Is the following statements true of false? (prove or disprove)

$$\forall x \in \mathbb{R} \exists y \in \mathbb{R} (y^2 > x) \to (x < 0).$$

Problem 10 Prove that

$$\lim_{n \to \infty} \frac{\sin n^2}{n+3} = 0.$$

Problem 11 Prove that for all real numbers a, b, $a < b$,
$(a,b) \cap \mathbb{Q} \neq \emptyset$.

Problem 12 Prove that

$$\sum_{i=1}^{n} (2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

Problem 13 Negate the following:

$$\forall \epsilon > 0 \quad \exists \delta > 0 \quad \forall x, y \in \mathbb{R} \quad (0 < |x-y| < \delta \to |e^x - e^y| < \epsilon).$$

Problem 14 Let X be a finite nonempty set. Prove that the number of subsets of X with even number of elements is equal to the number of subsets of X with odd number of elements.

Problem 15 Prove that a subset of a countable set is countable. (from the definition)

Problem 16 Prove that the number of triangles in the Euclidean plane with vertices having integer coordinates is countable. (results proven in class can be used)