TOPICS of MATH 307

Matrices

-Multiplication of matrices
-Special matrices (diagonal, upper(lower) triangular etc.)
-Transpose and inverse of a matrix
-Determinants (definition, calculation, properties)
-Rank of a matrix
-Relationship between rank, determinant and singularity of a matrix

Systems of Linear Algebraic Equations

-Matrix of a system
-Augmented matrix of a system
-Solution of a system
-Gauss-Jordan elimination
a)Elementary row operations
b)Reduced Row Echelon Form
c)Leading ones (pivot columns)
d)Rank of a matrix

Subspaces of $\mathbb{R}^n$

-Definition (closure by addition and scalar multiplication)
-Linear independence
-Basis and dimension
-Geometric interpretation of 0, 1, 2, 3-dimensional subspaces
-Coordinate systems and transition matrices
-Finding orthonarmal bases using Gram-Schmidt method

Linear transformations

-Definition (linearity properties)
-Matrix of a linear transform (how to find it)
-Image and Kernel of linear transform
-Inverse of a linear transform
-Matrices of linear transforms in different bases
-Geometric applications of linear transforms: finding matrices of reflexions, projections, dilations, shears.

Eigenvalues and eigenvectors, diagonalization

-Definitions and calculations of eigenvalues, eigenvectors, eigenspaces
-Matrix and linear transform diagonalization (necessary conditions)

BASIC PROBLEMS Let $L(\mathbf{x}) = A\mathbf{x}$.
Calculate (for each matrix given below)

a)$det(A)$
b)$rref(A), rank(A)$
c)A^{-1}(or show that it is singular)
d)eigenvalues, eigenspaces of A
e)diagonalize A if possible and find A^{103}
f)$Ker(L), Im(L)$, (sketch these subspaces) $dim(Ker(L))$, $dim(Im(L))$
g) all solutions of $L(\mathbf{x}) = (1, 2, 3, -1)$.
h)a matrix of L in a new basis $(1, 2, 0, 1), (0, 0, 2, 3), (1, 0, 0, 0), (0, 0, 0, 1))$.
i)an orthonormal basis for $Ker(L)$ and $Im(L)$.

Do parts d)e) for matriced marked $*$ only.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 3 & 1 & -1 \\ 0 & 5 & 6 & 0 \end{pmatrix}. \quad (*)A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$(*)A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (*)A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix}.$$