Mathematics 503 Problem Set 4

Due Friday, April 18, 2003

1. (Refer to **Exercise 6.1** on page 305.) Use one of MATLAB's stiff solvers to compute the solution of the given system with the stated initial values on the interval $0 \le t \le 4 \cdot 10^7$. Display a log-log plot showing all three solution components against t.

Recall from class that the component x_2 is much smaller than the others, so you must choose appropriate error tolerances for the three components to ensure an accurate solution.

Singular perturbation theory predicts that the solution is well approximated by

$$x_1(t) = x(t) \qquad x_2(t) = \epsilon z(t) \qquad x_3(t) = (1 - x(t) - \epsilon z(t))$$

in which $x(t)$ and $z(t)$ are the solution of the *reduced problem*

$$\begin{aligned} x' &= -ax + z(1 - x - \epsilon z) \qquad x(0) = 1 \\ 0 &= ax - z(1 - x - \epsilon z) - bz^2 \end{aligned}$$

using $a = 1/25$, $b = 3/10$ and $\epsilon = 10^{-4}$.

Use one of MATLAB'S non-stiff solvers to compute the solution of the reduced problem and compare it with the solution of the full system. Are there any features of the original system that it misses?

2. The TR-BDF2 method in MATLAB is an implicit Runge-Kutta formula that has the Butcher array

$$\begin{array}{c|ccc} 0 & & & \\ 2\alpha & \alpha & \alpha & \\ 1 & b_1 & b_2 & \alpha \\ \hline & b_1 & b_2 & \alpha \end{array}$$

with $\alpha = 1 - \frac{1}{2}\sqrt{2}$, $b_1 = b_2 = \frac{1}{4}\sqrt{2}$. Show that the stability function for this formula (see page 251)

$$E(z) = 1 + zb^T(I - zA)^{-1}e$$

satisfies $|E(z)| < 1$ for Re $z < 0$, i.e. verify that the formula is A-stable. [HINT: $E(z)$ is a rational function with no poles in the left half plane, so by the maximum modulus theorem it suffices to show that $|E(iy)|^2 = E(iy) \cdot E(-iy)$ is bounded by 1 for $0 \le y < \infty$.]

3. Do **Exercise 7.4**, pages 384–385.