

Mathematics 502 Problem Set 3

Due Monday, November 4, 2002

By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.

1. Answer QUESTION 4.13 of Demmel.

2. In \mathbf{R}^3 , the orthogonal complement of any plane is one-dimensional, spanned by any nonzero vector normal to the plane. Hence if two planes have unit normal vectors \mathbf{n}_1 and \mathbf{n}_2 we can define the *angle between the planes* to be the angle between their normals, determined by $\cos \theta = \mathbf{n}_1 \cdot \mathbf{n}_2$.

Let A be a *symmetric* 3×3 matrix with eigenvalues $|\lambda_1| > |\lambda_2| > |\lambda_3|$. Let \mathbf{z}_1 and \mathbf{z}_2 be orthonormal vectors, and assume that the plane that they span does not contain an eigenvector for λ_3 .

Let one step of subspace iteration be performed starting with $[\mathbf{z}_1 \mathbf{z}_2]$, producing orthonormal vectors $[\mathbf{q}_1 \mathbf{q}_2]$. Show that the new plane makes a smaller angle with the (λ_1, λ_2) eigenspace than the old one did. If the angle at iteration n is θ_n , does $\theta_n \rightarrow 0$? If so, what is its rate of convergence?

3. Solve Problem 15 in the §4.4 of Dennis & Schnabel, page 84. Along the way, show that a quadratic function is equal to any of its quadratic models: that is, for any $x_0 \in \mathbf{R}^n$,

$$f(x) = f(x_0) + \nabla f(x_0)^T(x - x_0) + \frac{1}{2}(x - x_0)^T \nabla^2 f(x_0)(x - x_0).$$