

# Mathematics 502 Problem Set 2

Due Monday, October 7, 2002

**By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.**

Hand in all analytical work that contributes to your solution. Submit printed copies of all MATLAB scripts (except M-files provided with the textbook), output tables, and graphs. Use the `diary` feature to get printed output from MATLAB sessions. Be sure to

- (a) Exploit vectorization whenever possible.
- (b) Include explanatory comments in all programs.
- (c) Make informative labels for all tables and graphs.
- (d) Edit output to make it compact and easy to read.

1. In a 1994 baseball game Atlanta Braves second baseman Mark Lemke got 5 hits in 6 times at bat. The announcer reported that this raised his batting average from .182 to .210. How many hits did he have in how many times at bat before the game started?

A player's *batting average* is computed by dividing the number of hits by the number of times at bat and rounding the result to three decimal places. Writing  $R_3$  for the operation of rounding to three places,  $y$  for the pregame number of times at bat and  $x$  for the pregame number of hits, we can describe the situation by the equations

$$R_3\left(\frac{x}{y}\right) = .182 \quad R_3\left(\frac{x+5}{y+6}\right) = .210 \quad (1)$$

and your task is to find the solution in positive integers  $x$  and  $y$ .

If you just ignore the rounding operation you do not get integer solutions. Show this by rewriting the equations

$$\frac{x}{y} = .182 \quad \frac{x+5}{y+6} = .210 \quad (2)$$

as a linear system and solving with MATLAB. Observe further that you do not get a solution of the rounded system (1) by rounding a solution of (2).

Instead, solve (1) this way: find all points with integer coordinates that lie between the two lines

$$\frac{x}{y} = .182 \pm .0005$$

and between the two lines

$$\frac{x + 5}{y + 6} = .210 \pm .0005$$

by plotting a graph and inspecting the region of overlap. Look carefully: there may be more than one solution!

Finally, observe that each solution point you find is the solution of a linear system that is “nearby” the system (2). Write the system (2) as  $Au = b$  with  $u$  representing the vector  $[x; y]$ . Each point you find in your plot is the solution of a system  $(A + \Delta A)(u + \Delta u) = b + \Delta b$ . Make a table showing  $\|\Delta A\|/\|A\|$ ,  $\|\Delta b\|/\|b\|$ , and  $\|\Delta u\|/\|u\|$ . Is the relative change in  $u$  accounted for sufficiently by the relative changes in  $A$  and  $b$  and the condition number of  $A$ ?

**2.** Let  $A$  be a symmetric positive definite *tridiagonal* matrix. Then the Cholesky factorization of  $A = LL^T$  yields a *bidiagonal* matrix  $L$ .

- (a) Suppose  $A$  is  $n \times n$ . Devise a scheme to represent  $A$  and  $L$  by two  $n$ -component vectors each.
- (b) Invent a general tridiagonal Cholesky algorithm that uses your representation of  $A$  from part (a) to compute  $L$ , represented also as in part (a).
- (c) Invent a general algorithm to solve  $LL^T x = b$  when  $L$  is a nonsingular lower bidiagonal matrix.
- (d) Use your algorithms from parts (b) and (c) to solve the  $64 \times 64$  system in which  $A$  is the matrix with 2 on the main diagonal and  $-1$  on the sub- and super-diagonals, and  $b$  is the vector with all components equal to 1.

**3.** Answer QUESTION 3.15, page 136 of Demmel.