Mathematics 502 Final Exam

By submitting a paper for this exam you declare that you are not submitting any unattributed work of any other person.

1. Dennis & Schnabel discuss on pages 56–57 an algorithm for computing the QR factorization of a matrix $A_+ = A_c + uv^T$ given the QR factorization of A_c.

If M is an $n \times n$ matrix, and $J = J(i, j, \alpha, \beta)$ a Jacobi rotation, show how the product JM can be computed with the minimum number of floating point operations.

"The reader can verify," Dennis & Schnabel declare, "that the entire QR update process requires only $O(n^2)$ operations." Indeed, show that the algorithm requires $Cn^2 + O(n)$ flops, find the constant C, and verify that the "most expensive part is getting $Q_+ = Q_c\tilde{Q}$ collected to save for the next step."

2. Consider QUESTION 2.13, part 1, page 95 of *Demmel*.

You saw the proof of the Sherman-Morrison formula in class. If $A + uv^T$ is singular, what is the null space? (In your answer, determine the dimension and find a basis.)

Prove the Sherman-Morrison-Woodbury formula.