Mathematics 471 Problem Set 1

Due Thursday, January 25, 2001

By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.

Hand in all analytical work that contributes to your solution. Submit printed copies of all programs (except those provided with the textbook), output tables, and graphs. (Use the `diary` feature to get printed output from MATLAB sessions.) Be sure to

(a) Include explanatory comments in all programs, and make informative labels for all tables and graphs.

(b) Edit output to make it compact and easy to read.

1. Consider the quadratic equation $x^2 + (2 \times 10^8)x + 1 = 0$. Let us stipulate that the roots are given accurately for IEEE long precision by the MATLAB fragment

```
>> a = 1; b = 2e8; c = 1;
>> format long e
>> roots([a b c])
```

Now calculate the roots (using IEEE long precision) using both formulas (4) and (5) in the article by Goldberg. Determine, for each computed value of each root, the absolute and relative error (taking those computed by the MATLAB `roots` function as the “exact” roots), and the residual. Display your results in a neatly labelled table.

Predict, from properties of IEEE arithmetic, the smallest value of b for which $\sqrt{b^2 - 4ac}$ evaluates to b exactly. Is your prediction accurate for the combination of programming language and precision you are using?

2. Let

$$\begin{aligned} f(x,p) &= x^8 - (36+p)x^7 + 546x^6 - 4536x^5 + 22449x^4 - \\ &\quad 67284x^3 + 118124x^2 - 109584x + 40320 \end{aligned}$$

Then f is a polynomial in x of degree eight, depending on p as a parameter. Observe that $f(x,0) = (x-1)(x-2)\ldots(x-8)$, so that when $p = 0$ the roots of f are $1, 2, \ldots, 8$. For variable p, the eight roots of $f(x,p)$ may be regarded as functions of p: that is, for each $i = 1, \ldots, 8$ we have a root $f(x_i, p) = 0$ with $x_i = x_i(p)$ and $x_i(0) = i$. For each root x_i, compute $\left.\frac{dx_i}{dp}\right|_{p=0}$ (use implicit differentiation). According to this measure, which root is most sensitive to the perturbation?

Find the roots of $f(x, 0.1)$. Does the movement of the roots bear out your sensitivity calculation?

3. Investigate the fixed point iteration $x_{n+1} = g(x_n)$ for

$$g(x) = -4 + 4x - \frac{1}{2}x^2.$$

(a) Solve $g(x) = x$ and show that $P = 2$ and $P = 4$ are fixed points.

(b) Use the starting value $p_0 = 1.9$ and compute p_1, p_2 and p_3.

(c) Use the starting value $p_0 = 3.8$ and compute p_1, p_2 and p_3.

(d) Show how the behavior of these two iterations can be predicted by analysis of the contraction (or expansion) rate at each fixed point.