Math 471 Midterm Exam

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

1. The table below indicates that the Bessel function $J_0(x)$ has a root in the interval $[2.4, 2.5]$. Using only the information in the table, find the best

x	$J_0(x)$	$J_0'(x)$
2.4	0.00250 76833	-0.52018
2.5	-0.04838 37764	-0.49709

approximation to the root obtainable by one step of

a. the bisection method;

b. the secant method;

c. Newton's method.

2. In the text, pages 174–175 display a matrix A and vector b in **(1)**, and matrices L and U with $L \cdot U = A$ in **(2)** and **(3)**, respectively. Use forward and back substitution with L and U to compute the solution of the linear system **(1)**.

[Exam Continues on Reverse]

3. Let B_n be the bidiagonal matrix with 1 on the main diagonal, -2 on the next diagonal above, and 0 everywhere else. For example B_4 is

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. Count the number of floating point operations required to solve a linear system $B_n x = b$, as a function of n.

b. Determine the last column of B_{52}^{-1}. [HINT: Write e_{52} for the vector of 51 zeros followed by a one. The desired vector is the solution of $B_{52}x = e_{52}$. Each matrix B_n is upper triangular.]

c. Use your answer from (b) and the fact that $\|B_{52}^{-1}\| \geq \|B_{52}^{-1}e_{52}\|$ to estimate how big the condition number $\kappa(B_{52})$ is, in any convenient norm.

4. Consider the equation

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0. \tag{1}$$

a. Show that (1) is equivalent to

$$x = g(x) = 10 - 35/x + 50/x^2 - 24/x^3.$$

b. Show that the fixed-point iteration $x_{n+1} = g(x_n)$ is contractive near $x = 4$.

[Exam Continues on Obverse]