

## MATH 317: EXERCISES ON LINEAR INDEPENDENCE

### 1. LINEAR DEPENDENCE AND LINEAR INDEPENDENCE

**Definition.** Let  $\mathcal{S}$  be a subset of a vector space  $\mathcal{V}$ . Then  $\mathcal{S}$  is **linearly dependent** if some vector in  $\mathcal{S}$  can be expressed as a linear combination of the other vectors in  $\mathcal{S}$ .

The set  $\mathcal{S}$  is **linearly independent** if  $\mathcal{S}$  is not linearly dependent.

### 2. SOME EXERCISES

- (1) Express the definition of linear dependence in symbolic form. Use a quantifier and the concept of span.
- (2) Express the *negation* of “ $\mathcal{S}$  is linearly dependent” in symbolic form. (The negation is, of course, another way to say “ $\mathcal{S}$  is linearly independent.”) Rewrite your symbolic statement as an English sentence.
- (3) Is the empty set linearly dependent?
- (4) Is the one-element set  $\mathcal{S} = \{\mathbf{0}\}$  linearly dependent?
- (5) If  $\mathbf{v} \neq \mathbf{0}$ , is the one-element set  $\mathcal{S} = \{\mathbf{v}\}$  linearly dependent?
- (6) Prove: Any set  $\mathcal{S}$  with  $\mathbf{0} \in \mathcal{S}$  is linearly dependent.
- (7) Suppose a two-element set  $\mathcal{S} = \{\mathbf{u}, \mathbf{v}\}$  is linearly dependent. Must  $\mathbf{u}$  and  $\mathbf{v}$  be scalar multiples of each other?
- (8) Let  $\mathcal{S} = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  be a three-element set. Suppose no element of  $\mathcal{S}$  is a scalar multiple of any other. Must  $\mathcal{S}$  be linearly independent?