

## MATH 317: EXERCISES ON LINEAR INDEPENDENCE

### 1. LINEAR DEPENDENCE AND LINEAR INDEPENDENCE

**Definition.** The (indexed) set of vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  is **linearly independent** if whenever  $c_1, \dots, c_k$  are coefficients such that  $c_1\mathbf{v}_1 + \dots + c_k\mathbf{v}_k = \mathbf{0}$ , then  $c_1 = \dots = c_k = 0$ .

The set of vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  is **linearly dependent** if it is not linearly independent.

### 2. SOME EXERCISES

- (1) Express the definition of linear independence in symbolic form. Use quantifiers.
- (2) Express the *negation* of “ $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  is linearly independent” in symbolic form. (The negation is, of course, another way to say “ $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  is linearly dependent.”) Rewrite your symbolic statement as an English sentence.
- (3) Is the empty set linearly independent?
- (4) Is the one-element set  $\{\mathbf{0}\}$  linearly independent?
- (5) If  $\mathbf{v} \neq \mathbf{0}$ , is the one-element set  $\{\mathbf{v}\}$  linearly independent?
- (6) Prove: Any set  $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  containing  $\mathbf{0}$  is linearly dependent.
- (7) Suppose a two-element set  $\{\mathbf{u}, \mathbf{v}\}$  is linearly dependent. Must  $\mathbf{u}$  and  $\mathbf{v}$  be scalar multiples of each other?
- (8) Let  $\mathcal{S} = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  be a three-element set. Suppose no element of  $\mathcal{S}$  is a scalar multiple of any other. Must  $\mathcal{S}$  be linearly independent?