Mathematics 273 Problem Set 4

Due Wednesday, November 14, 2001

By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.

Hand in all analytical work that contributes to your solution. Submit printed copies of all MATLAB scripts (except M-files provided with the textbook), output tables, and graphs. Use the `diary` feature to get printed output from MATLAB sessions. Be sure to

(a) Exploit vectorization whenever possible.

(b) Include explanatory comments in all programs.

(c) Make informative labels for all tables and graphs.

(d) Edit output to make it compact and easy to read.

1. In a certain (fictitious) electronic device, the current-voltage characteristic is given by the ideal diode equation

$$i = I_S(T)\left(e^{qv/kT} - 1\right).$$

Here i is current in amperes, q is the electron charge (1.6×10^{-19} coulomb), v is voltage, k is Boltzmann's constant (1.38×10^{-23} joule/degree Kelvin), and T is temperature in degrees Kelvin (= degrees Celsius +273.16). The factor $I_S(T)$ is the saturation current, a function of temperature and of the doping of the chips during the manufacturing process. For the device in question, the saturation current function is

$$I_S(T) = i_s e^{-\left(\frac{R-T}{10}\right)^2},$$

i_s is a constant depending on the doping; R is room temperature, 293.16 K.

(a) Create a MATLAB M-file `function i=idiode(T,v,is)` that computes the current. (The physical constants q, k, and R can be "hard-coded" into your function; the quantities T, v and i_s, which depend on operating conditions or the specific device, are function arguments.)

(b) To operate properly, this device must deliver a current of at least one amp when there is a potential difference v of 1 volt across it. Use `fzero` to find the two temperatures at which $i = 1$ when $v = 1$ and $i_s = 10^{-13}$. Express these temperatures in degrees Fahrenheit.

(c) Between the two temperatures you found in part (b) is one at which the current is a maximum. Use `fmin` to find this temperature and the corresponding maximum current (use the same values of v and i_s as before).

2. The electrostatic potential φ satisfies *Laplace's equation*

$$-\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2} - \frac{\partial^2\varphi}{\partial z^2} = -\nabla^2\varphi = 0$$

in regions that are free of electric charge. In the region between two concentric spheres whose potential is known, the potential is spherically symmetric, so we transform Laplace's equation to spherical coordinates and get (since $\varphi = \varphi(r)$)

$$
\begin{aligned}
-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right) &= 0 \qquad R_1 < r < R_2 && (1)\\
\varphi(R_1) &= V_1 && (2)\\
\varphi(R_2) &= V_2. && (3)
\end{aligned}
$$

Here R_1 and R_2 are the respective radii of the inner and outer spheres, and V_1 and V_2 are their known potentials. Equation (1) is called the *radial Laplace equation*.

(a) Determine analytically the exact solution φ of the radial Laplace equation, for the data $R_1 = 4, R_2 = 5, V_1 = 5, V_2 = 1$.

(b) Set up the divided difference scheme of §**7.3.1** (as corrected in the class notes!) for (1) and write out explicitly the coefficients of the resulting linear system for the case $n = 7$.

(c) Write a MATLAB script that does the following:

- Set up the difference equations and solve using the tridiagonal Cholesky method of §**7.3.2**. Do the cases $n = 7, 13, 25$.
- In one figure window, show the exact solution as a solid curve, and the points of your computed solution with `'+'`.
- In a second figure window, superpose three plots (one for each mesh size) of the scaled error

$$\frac{\left|\varphi_{\text{computed}} - \varphi_{\text{exact}}\right|}{h^2}$$

as a function of r. (The difference scheme is second order accurate, i.e. the error is $\mathcal{O}(h^2)$, so the quantity plotted here is theoretically converging to a limit as $h \to 0$.)

3. Write a MATLAB function M-file

```
function x = fmnewtn(fobj,fgradhess,x0,xtol,gtol,maxit)
```

that minimizes multivariate functions by Newton's method.

The first argument to `fmnewtn` is the string that names the objective function M-file; the second argument `fgradhess` is a string that names the function M-file for computing both the gradient and Hessian of the objective function. Then `x0` is the initial guess for a minimizer, `xtol` is the x tolerance, `gtol` is the tolerance for the gradient and `maxit` is the maximum number of iterations allowed.

Use `fmnewtn` to minimize the five-variable function

$$
\begin{aligned}
z &= 0.5(x_1^4 - 16x_1^2 + 5x_1) + 0.5(x_2^4 - 16x_2^2 + 5x_2)\\
&\quad +(x_3 - 1)^2 + (x_4 - 1)^2 + (x_5 - 1)^2
\end{aligned}
$$

Start with $x_1 = 1$, $x_2 = 2$, $x_3 = 0$, $x_4 = 2$, $x_5 = 3$, and iterate until either the change in x or the norm of the gradient is smaller than `1e-7`.