Mathematics 273 Problem Set 3

Due Friday, October 26, 2001

By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.

Hand in all analytical work that contributes to your solution. Submit printed copies of all MATLAB scripts (except M-files provided with the textbook), output tables, and graphs. Use the `diary` feature to get printed output from MATLAB sessions. Be sure to

(a) Exploit vectorization whenever possible.

(b) Include explanatory comments in all programs.

(c) Make informative labels for all tables and graphs.

(d) Edit output to make it compact and easy to read.

1. The system of equations for an electric ladder is given by

$$\begin{aligned} 10x_1 - 5x_2 &= 20 \\ -5x_{i-1} + 10x_i - 5x_{i+1} &= 0, \qquad i = 2, 3, \ldots, N-1 \\ -5x_{N-1} + 10x_N &= 0. \end{aligned}$$

Find the solution in the cases $N = 10$ and $N = 20$. Your method should be as efficient as possible both in the use of memory and in the number floating point operations required.

2. In a 1994 baseball game an Atlanta Braves player got 5 hits in 6 times at bat. The announcer reported that this raised his batting average from .182 to .210. How many hits did he have in how many times at bat before the game started?

A player's *batting average* is computed by dividing the number of hits by the number of times at bat and rounding the result to three decimal places. Writing R_3 for the operation of rounding to three places, y for the pregame number of times at bat and x for the pregame number of hits, we can describe the situation by the equations

$$R_3\left(\frac{x}{y}\right) = .182 \qquad R_3\left(\frac{x+5}{y+6}\right) = .210 \tag{1}$$

and your task is to find the solution in positive integers x and y.

If you just ignore the rounding operation you do not get integer solutions. Show this by rewriting the equations

$$\frac{x}{y} = .182 \qquad \frac{x+5}{y+6} = .210 \tag{2}$$

as a linear system and solving with MATLAB. Observe further that you do not get a solution of the rounded system (1) by rounding a solution of (2).

Instead, solve (1) this way: find all points with integer coordinates that lie between the two lines

$$\frac{x}{y} = .182 \pm .0005$$

and between the two lines

$$\frac{x+5}{y+6} = .210 \pm .0005$$

by plotting a graph and inspecting the region of overlap. Look carefully: there may be more than one solution!

Finally, observe that each solution point you find is the solution of a linear system that is "nearby" the system (2). Write the system (2) as $Au = b$ with u representing the vector $[x; y]$. Each point you find in your plot is the solution of a system $(A + \Delta A)(u + \Delta u) = b + \Delta b$. Make a table showing $\|\Delta A\|/\|A\|$, $\|\Delta b\|/\|b\|$, and $\|\Delta u\|/\|u\|$. Is the relative change in u accounted for sufficiently by the relative changes in A and b and the condition number of A?

3. Retrieve the table of the index of refraction data for fused quartz glass from `Examples/Chapter_7/refraction.dat`. Perform a least squares fit with polynomials of degrees 2 through 7. Plot all six polynomials in separate subplots of a single figure window, showing each polynomial curve with the original data points. In a second figure window, plot the norm of the residual vector against the polynomial degree.

Use both the appearance of the graphs and the size of the residuals to determine which is the best polynomial to use, and defend your choice. (Recall that the interpolating polynomial of degree 8 computing in the previous assignment fits the data *exactly*, but is defective because of oscillations between data points.)