

## Mathematics 273 Problem Set 2

Due Wednesday, October 3, 2001

**By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.**

Hand in all analytical work that contributes to your solution. Submit printed copies of all MATLAB scripts (except M-files provided with the textbook), output tables, and graphs. Use the `diary` feature to get printed output from MATLAB sessions. Be sure to

- (a) Exploit vectorization whenever possible.
- (b) Include explanatory comments in all programs.
- (c) Make informative labels for all tables and graphs.
- (d) Edit output to make it compact and easy to read.

1. (Refer to Problem **P1.5.1**) If  $f$  is a function,  $a$  a point, and  $h$  an increment, the *centered difference approximation* to  $f'(a)$  (with increment  $h$ ) is

$$C_h = \frac{f(a+h) - f(a-h)}{2h}.$$

- (a) Show that if  $|f^{(3)}(x)| \leq M_3$  then  $|C_h - f'(a)| \leq \frac{M_3}{6}h^2$ . (Use the Taylor polynomial for  $f$  of order 2 at  $a$ , with remainder.)
- (b) Modeling the error in the evaluation of  $C_h$  by

$$\text{errC}(h) = \frac{M_3 h^2}{6} + 2\frac{\delta}{h},$$

find the  $h$  that makes this estimated error a minimum. This is the optimum  $h$  for the centered difference approximation.

- (c) Revise the function `Derivative.m` to accept as a fifth argument an estimate  $M_3$  of the magnitude of  $f^{(3)}$  and deliver the more accurate of  $D_h$  and  $C_h$ , by filling in the skeleton below.

```
function [d,err] = Derivative(fname,a,delta,M2,M3)
% Explain arguments
% ...
% Supply defaults if called with 4, 3 or 2 arguments
% ...
% Estimate minimum error for D_h and C_h
% ...
% Using the formula with smaller estimated minimum error:
err = Estimated minimum error
h   = Optimum h
d   = Approximate derivative
```

Use 100 for the default values of `M2` and `M3`. Use what you learned about roundoff error in §1.4.4 to supply an appropriate default value for `delta`.

- (d) Demonstrate the performance of your function `Derivative` by estimating the derivative of the Bessel function  $Y_{10}$  at the points `1:20`. (You evaluate this function with `y=bessely(10,x)`. Do `help bessely` for further information.) Print a table showing the derivative value and error estimate at each point.

2. The following table shows the refractive index  $\mu$  for fused quartz glass measured as a function of the wavelength of light  $\lambda$  in microns:

$\lambda$	0.35	0.40	0.45	0.50	0.55	0.65	0.70	0.80	0.90
$\mu(\lambda)$	1.477	1.470	1.466	1.462	1.459	1.457	1.455	1.453	1.452

Find the interpolating polynomial for these data, expressed in both the Vandermonde and Newton forms. Plot the interpolants (as continuous curves) on a dense set of points spanning the data, and show the data points as `'*'`. Mathematically, the interpolants are identical. How does the difference between the computed interpolants compare to roundoff?

Use the interpolant to estimate  $\mu(0.43)$  and  $\mu(0.86)$ . Do you believe these estimates? Explain!

3. (Refer to Problem **P5.3.3**.) Write a MATLAB function `[A,b]=Laplace(m,n)` that returns the *matrix*  $A$  and *right side vector*  $b$  for the system of equations described in the problem statement, for general  $m$  and  $n$ .

(You will need to correct the MATLAB command setting temperatures on the north boundary: `fnorth = sin(pi/2*x) .* exp(-x)`;) )

To determine the right-side vector  $b$ , notice that the right hand side of an equation is 0 unless the '+' point the equation corresponds to is *adjacent* to an 'o' point on the north or south boundary of the rectangle. In that case, the right hand side is just the temperature at that 'o' point.

Print  $A$  and  $b$  for the case  $m = 3, n = 4$ . Observe that the matrix  $A$  for this case has the block structure

$$A = \begin{bmatrix} M & -I & 0 \\ -I & M & -I \\ 0 & -I & M \end{bmatrix}$$

in which  $I$  is the  $4 \times 4$  identity matrix, 0 represents a  $4 \times 4$  zero matrix, and  $M$  is the tridiagonal matrix

$$M = \begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix}.$$

You can use colon notation to construct  $A$  from block matrices for any  $m, n$ .

Use `\` to solve the linear system  $At = b$  in the cases ( $m = 3, n = 4$ ) and ( $m = 8, n = 14$ ). In the latter case, do `cs = contour(Tmatrix,10); clabel(cs)`; as called for in the text; to arrange the solution vector  $t$  as an array `Tmatrix`, you may find the commands `reshape` and `flipud` helpful.