Mathematics 273 Problem Set 1

Due Friday, September 14, 2001

By submitting a paper for this assignment you declare that you are not submitting any unattributed work of any other person.

Hand in all analytical work that contributes to your solution. Submit printed copies of all MATLAB scripts (except M-files provided with the textbook), output tables, and graphs. Use the `diary` feature to get printed output from MATLAB sessions. Be sure to

(a) Exploit vectorization whenever possible.

(b) Include explanatory comments in all programs.

(c) Make informative labels for all tables and graphs.

(d) Edit output to make it compact and easy to read.

1. A half-rectified sine wave has the formula

$$\sin^{(+)}(x) = \frac{1}{2}(\sin x + |\sin x|).$$

Imitating the `SquareWave` demonstration from class, find the harmonic decomposition of the half-rectified sine wave and create a MATLAB M-file called `HalfRectSin.m` to display it with appropriate plots, as follows.

(a) (Analytical preparation.) The harmonic decomposition of $\sin x$ is, obviously, $\sin x$. Since $|\sin x|$ is an even function, it can be analyzed into cosines in the form

$$|\sin(x)| = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \ldots.$$

The amplitudes a_k, $k = 1, 2, 3, \ldots$ are given by

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin(x)| \cos kx \; dx = \frac{2}{\pi}\int_{0}^{\pi} \sin x \cos kx \; dx.$$

Find the general coefficient a_k in simplest form.

(b) Calculate the successive partial sums that include, one by one, the first six nonvanishing cosine components of $\sin^{(+)}(x)$. Then plot two figures. In Figure 1 show one period of the half-rectified sine wave with the first six partial sums superimposed; in Figure 2 show the half-rectified sine wave with each partial sum in a separate subplot.

2. The Parking Problem. Cars that are one unit long arrive and attempt to park at random points along a street of length $N = 10$. A car can "park" if there is at least one half unit of space on either side of its center. (Neglect the maneuvering space that would be needed for parallel parking.) What is the average number of cars that can park before no more cars will fit?

An experiment consists of attempting to park cars one at a time at random points until no more cars will fit. Repeat the experiment 50 times and plot the cumulative fraction of space occupied by parked cars.

HINT: Whether a car will fit at a given spot, and whether the street is full after a new car has been parked, can each be determined by a single vectorized MATLAB statement.

3. Show that the formula for the roots of a quadratic equation $ax^2+bx+c = 0$ ("quadratic formula")

$$x = \frac{-b \pm \sqrt{D}}{2a} \qquad D = b^2 - 4ac$$

is mathematically identical to

$$x = \frac{2c}{-b \mp \sqrt{D}} \qquad D = b^2 - 4ac.$$

Consider next the quadratic equation $x^2 + (3 \times 10^8)x + 2 = 0$. Let us stipulate that MATLAB finds the roots accurately when you apply the `roots` function to the coefficient array `[1 3e8 2]`. Do it, and call the results the "exact" roots.

Now use the quadratic formula in both of the forms above to calculate the roots. (This yields two "computed" values for each root.) For each computed value of each root, determine the absolute error, the relative error and the residual. Display your results in a neatly labelled table.

What happens? Why do the mathematically identical formulas give numerically different results?