1. Form the augmented matrix, and subtract $2 \times \text{Row 1}$ from Row 2 to obtain
\[
\begin{bmatrix}
1 & 1 & 1 + \delta \\
0 & \delta & -2 \delta
\end{bmatrix}.
\]
Then back-substitution gives $x_2 = -2$, $x_1 = 3 + \delta$ for the exact solution.

The error and residual vectors are
\[
\hat{x} - x = \begin{bmatrix} -2 - \delta \\ 2 \end{bmatrix} \quad A\hat{x} - b = \begin{bmatrix} -\delta \\ 0 \end{bmatrix}.
\]

The fact that a vector with such a large error has a tiny residual indicates that the matrix is nearly singular. The nearby matrix
\[
\hat{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}
\]
is exactly singular (its columns are linearly dependent) and its relative distance to $A$, $\|A - \hat{A}\|_\infty / \|A\|_\infty = \delta / (4 + \delta)$ is extremely small.

2. (a) The $\text{polyfit}$ function is unsuitable because we are looking for a linear function with a zero intercept, but $\text{polyfit}$’s linear fit will have the form $w = c_1 t + c_2$.

(b) Variables in the standard form are $A = t$, $b = w$ and $x = c$.

(c) Using the formulas on p. 249 gives the matrix
\[
G_1 = \frac{1}{5} \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}
\]
for rotating in the 2–3 plane. Applying the rotation produces
\[
t' = \begin{bmatrix} 60 \\ -45 \\ 0 \end{bmatrix}, \quad w' = \begin{bmatrix} 435 \\ -295 \\ -25 \end{bmatrix}.
\]

Applying next the rotation
\[
G_2 = \frac{1}{5} \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}
\]
in the 1 − 2 plane converts $t$ to the triangular form.

$$R = \begin{bmatrix} 75 \\ 0 \\ 0 \end{bmatrix}, \quad w'' = \begin{bmatrix} 525 \\ 25 \\ -25 \end{bmatrix}.$$  

So $c = 525/75 = 7$ and the norm of the minimum residual is $25\sqrt{2}$.

(d) The command $c = t \setminus w$ computes $c$.

3. We learned in §1.5 and the homework problem associated with it that a good choice of $\delta_c$ would be

$$\delta_c = 2\sqrt{\text{eps} \cdot F/M_2}$$

with $F = |f(x_c)|$ and $M_2$ a bound for $f''$ near $x_c$. Obviously $F$ is available, but without prior knowledge of $f$, $M_2$ is not and you have to make some reasonable guess. If the actual $M_2$ is very much larger than your guess, then the computed derivative can have large errors, as was seen in the homework problem.

4. One solution (as used in fmin and demonstrated in example script sqi.m) is to use parabolic interpolation: find the parabola of the form

$$y = c_1 + c_2(x - 4.4) + c_3(x - 4.4)(x - 4.5)$$

that fits the data in the table; approximate the minimum of the function by the vertex of the parabola.

Calculating according to page 84 gives

$$c_1 = -0.21627320$$
$$c_2 = \frac{-0.21722892 - (-0.21627320)}{4.5 - 4.4} = -0.0095572$$
$$c_3 = \frac{-0.21722892 - (-0.21627320 + -0.0095572(.2))}{(.2)(.1)} = 0.108243$$

The vertex of the parabola is at the point $x$ where $c_2 + c_3(2x - 8.9) = 0$, giving $x = 4.494147$.

5. Move the constants to the left side so that the equations have the form $F = 0$. Then the residual vector and Jacobian at $x_1 = 0$, $x_2 = 0$ are

$$\begin{bmatrix} -0.6 \\ -4.6 \end{bmatrix} \quad \begin{bmatrix} 1.4 & -1 \\ 2x_1 - 1.6 & -1 \end{bmatrix}$$

so we must solve the system

$$1.4s_1 - s_2 = 0.6 \quad (1)$$
$$-1.6s_1 - s_2 = 4.6 \quad (2)$$

and find $s_1 = -1.3333$, $s_2 = -2.4667$. 

2