Math 273 Midterm II

1. Form the augmented matrix, and subtract 2×Row 1 from Row 2 to obtain

$$\begin{bmatrix} 1 & 1 & 1+\delta \\ 0 & \delta & -2\delta \end{bmatrix}.$$

Then back-substitution gives $x_2 = -2$, $x_1 = 3 + \delta$ for the exact solution.

The error and residual vectors are

$$\hat{x} - x = \begin{bmatrix} -2-\delta \\ 2 \end{bmatrix} \qquad A\hat{x} - b = \begin{bmatrix} -\delta \\ 0 \end{bmatrix}.$$

The fact that a vector with such a large error has a tiny residual indicates that the matrix is nearly singular. The nearby matrix

$$\hat{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

is exactly singular (its columns are linearly dependent) and its relative distance to A, $\|A - \hat{A}\|_\infty / \|A\|_\infty = \delta/(4+\delta)$ is extremely small.

2.

(a) The `polyfit` function is unsuitable because we are looking for a linear function with a zero intercept, but `polyfit`'s linear fit will have the form $w = c_1 t + c_2$.

(b) Variables in the standard form are $A = t$, $b = w$ and $x = c$.

(c) Using the formulas on p. 249 gives the matrix

$$G_1 = \frac{1}{5}\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

for rotating in the $2-3$ plane. Applying the rotation produces

$$t' = \begin{bmatrix} 60 \\ -45 \\ 0 \end{bmatrix}, \qquad w' = \begin{bmatrix} 435 \\ -295 \\ -25 \end{bmatrix}.$$

Applying next the rotation

$$G_2 = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix}$$

in the $1-2$ plane converts t to the triangular form.

$$R = \begin{bmatrix} 75 \\ 0 \\ 0 \end{bmatrix}, \qquad w'' = \begin{bmatrix} 525 \\ 25 \\ -25 \end{bmatrix}.$$

So $c = 525/75 = 7$ and the norm of the minimum residual is $25\sqrt{2}$.

(d) The command `c = t\w` computes c.

3. We learned in §1.5 and the homework problem associated with it that a good choice of δ_c would be

$$\delta_c = 2\sqrt{\texttt{eps} \cdot F/M_2}$$

with $F = |f(x_c)|$ and M_2 a bound for f'' near x_c. Obviously F is available, but without prior knowledge of f, M_2 is not and you have to make some reasonable guess. If the actual M_2 is very much larger than your guess, then the computed derivative can have large errors, as was seen in the homework problem.

4. One solution (as used in `fmin` and demonstrated in example script `sqi.m`) is to use parabolic interpolation: find the parabola of the form

$$y = c_1 + c_2(x - 4.4) + c_3(x - 4.4)(x - 4.5)$$

that fits the data in the table; approximate the minimum of the function by the vertex of the parabola.

Calculating according to page 84 gives

$$\begin{aligned}
c_1 &= -0.21627320 \\
c_2 &= \frac{-0.21722892 - (-0.21627320)}{4.5 - 4.4} = -0.0095572 \\
c_3 &= \frac{-0.21722892 - (-0.21627320 + -0.0095572(.2))}{(.2)(.1)} = 0.108243
\end{aligned}$$

The vertex of the parabola is at the point x where $c_2 + c_3(2x - 8.9) = 0$, giving $x = 4.494147$.

5. Move the constants to the left side so that the equations have the form $F = 0$. Then the residual vector and Jacobian at $x_1 = 0$, $x_2 = 0$ are

$$\begin{bmatrix} -0.6 \\ -4.6 \end{bmatrix} \qquad \begin{bmatrix} 1.4 & -1 \\ 2x_1 - 1.6 & -1 \end{bmatrix}$$

so we must solve the system

$$\begin{aligned}
1.4s_1 - s_2 &= 0.6 \qquad (1) \\
-1.6s_1 - s_2 &= 4.6 \qquad (2)
\end{aligned}$$

and find $s_1 = -1.3333$, $s_2 = -2.4667$.