Math 273 Midterm II

2001 November 15

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

1. The vector $\hat{x} = (1, 0)^T$ is an approximate solution of the linear system $Ax = b$ with

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2+\delta \end{bmatrix} \qquad b = \begin{bmatrix} 1+\delta \\ 2 \end{bmatrix}$$

and $\delta = 10^{-14}$. Observe that A is nonsingular. Find the exact solution x.

(a) Find the error and the residual of $\hat{x}$. What about these vectors indicates that A is nearly singular?

(b) Find a singular matrix $\hat{A}$ that is near A. Using any convenient norm, calculate $\|A - \hat{A}\|/\|A\|$, the relative distance from A to $\hat{A}$.

2. It is desired to fit an equation of the form $w = ct$ to the data vectors

$$t = \begin{bmatrix} 60 \\ -27 \\ 36 \end{bmatrix} \qquad w = \begin{bmatrix} 435 \\ -197 \\ 221 \end{bmatrix}$$

by the method of least squares.

(a) Why can't MATLAB's `polyfit` function be used to solve this problem?

(b) Set up this problem in the standard form shown in the box on p. 241: what are the A, b and x?

(c) Find the QR factorization of your matrix A from part b. (Two rotations suffice.) Use the QR factorization of A to compute c and the minimum residual.

(d) Assuming data vectors `t` and `w` have been entered into MATLAB, give a one-line command that computes c.

[Exam continues on Reverse]

3. On page 289 Van Loan presents a "finite difference Newton framework." The accompanying code fragment says "`Choose delta`" but does not say how to choose this increment for the divided difference.

Explain how you would choose δ_c to maintain quadratic convergence, without any prior knowledge of the function f. What about f could cause your δ_c to be a poor choice?

4. The spherical Bessel function j_0 has a minimum at a point between 4.4 and 4.6. Using *only* the information in the table below, locate the minimum point as accurately as you can.

x	$j_0(x)$
4.4	−0.21627320
4.5	−0.21722892
4.6	−0.21601978

5. Carry out one step of Newton's method on the system

$$\begin{aligned} 1.4x_1 - x_2 &= 0.6 \\ x_1^2 - 1.6x_1 - x_2 &= 4.6 \end{aligned}$$

using the initial guess $x_1 = 0$, $x_2 = 0$.

[Exam begins on Obverse]