# Math 273 Midterm II

2000 November 9

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

**1.** Refer to the algorithm `CholTridSol` on page 259. Count the number of floating point operations (flops) this algorithm requires when its data are $n$-vectors $g$, $h$ and $b$.

Compare the flop count for the Cholesky algorithm for symmetric positive definite tridiagonal matrices (`CholTrid` and `CholTridSol` together) with that for the ordinary tridiagonal algorithm of §6.2. [It was shown in class that `CholTrid` requires $4n + \mathcal{O}(1)$ flops.] Which is more efficient?

**2.** Solve the linear system $Ax = b$ with

$$A = \begin{bmatrix} 12 & -4 \\ -5 & 6 \end{bmatrix}, \qquad b = \begin{bmatrix} 8 \\ 1 \end{bmatrix},$$

by finding a multiplier matrix $M$ so that $MA$ is upper triangular; then solve $MAx = Mb$ for x. (A multiplier matrix performs an elementary row operation on $A$.) With your solution, give also a unit lower triangular matrix $L$ and an upper triangular matrix $U$ so that $A = LU$.

**[Exam continues on Reverse]**

**3.** The electric current in an LR circuit is governed by the differential equation

$$L\frac{di}{dt} + Ri = E(t). \tag{1}$$

To find the values of $L$ and $R$ in a particular circuit, it is driven for 5 sec with a known voltage $E(t)$ and the current is measured at intervals of .05 sec yielding data vectors `i` and `E` of length 101.

(a) Give a MATLAB fragment, vectorized if possible, to compute from `i` a vector `didt` of estimates of $di/dt$ over the 5 sec interval.

(b) Give MATLAB commands to estimate $L$ and $R$ by the method of least squares: that is, by minimizing the sum of squares of residuals in Equation (1) at the data points. (Your commands should use only `i`, `didt`, `E`, and variables derived from them.)

**4.** Find the steepest descent vector $\mathbf{p}$ for the function

$$f(x_1, x_2) = 2x_1^2 + x_1 + x_2^2 - 2x_1x_2$$

at the point $\mathbf{x}_0 = (1, 1)^T$, and find the value of $\lambda$ for which $f(\mathbf{x}_0 + \lambda\mathbf{p})$ is a minimum.

**5.** Kepler's Equation is

$$E - e\sin E = t.$$

In this equation $e$ (eccentricity of an elliptical orbit) and $t$ (time, in units chosen so that $2\pi$ is one planetary year) are known and $E$ is to be determined.

Given $e = .2056$ and $t = 0.80$, find a good starting guess for $E$.

Apply one step of Newton's method to your starting guess to obtain a better approximate solution.