Math 273 Midterm I Solutions

1. A MATLAB command that creates a 2×128 array of dice is

```
>> Dice = ceil(6*rand(2,128));
```

The given code fragment counts the number of times in 128 throws that the sum shown on the dice is 8. It can be vectorized by

```
>> eights = sum(sum(Dice)==8)
```

2. Three approximations to $\Gamma'(1)$ using data in the table are

$$\frac{1-1}{1} \qquad \frac{\frac{1}{2}\sqrt{\pi}-1}{\frac{1}{2}} \qquad \frac{\frac{1}{2}\sqrt{\pi}-\sqrt{\pi}}{2\cdot\frac{1}{2}}$$

These are, respectively: forward difference with $h = 1$, forward difference with $h = 1/2$, and centered difference with $h = 1/2$. The last should be the most accurate, because the error in the centered difference is second order in h while the error in the forward difference is only first order. (In-class demonstration showed that roundoff error is not a significant factor with h's of this size.)

3. The cubic polynomial is

$$p(x) = c_1 + c_2(x-1) + c_3(x-1)(x-2) + c_4(x-1)(x-2)(x-3)$$

and the coefficients are determined from the data by

$$\begin{aligned}
x = 1, y = 1: \quad 1 &= c_1 \Rightarrow c_1 = 1 \\
x = 2, y = 1: \quad 1 &= 1 + c_2 \cdot 1 \Rightarrow c_2 = 0 \\
x = 3, y = 1: \quad 1 &= 1 + 0 \cdot 2 + c_3 \cdot 2 \cdot 1 \Rightarrow c_3 = 0 \\
x = 4, y = 25: \quad 25 &= 1 + 0 \cdot 3 + 0 \cdot 3 \cdot 2 + c_4 \cdot 3 \cdot 2 \cdot 1 \Rightarrow c_4 = 4.
\end{aligned}$$

4. The function on the left implements a row-oriented algorithm for computing the matrix-vector product. It uses dot-products, not saxpys, so the comments are not correct.

The function on the right implements a column-oriented algorithm that does use saxpys, so its description is correct as it stands.

5. The coding scheme in the M-file `SumOfSines.m` would be awkward if 25 harmonics were needed because one would have to type in all 25 columns of the matrix.

The matrix is easily set up with

```
>> A = sin( x * [1:m] );
```

as was used in the `SquareWave.m` demonstration M-file for Chapter 1.