

Math 273 Midterm I

2000 September 28

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

1. What do the following two MATLAB fragments do, if \mathbf{x} and \mathbf{y} are column vectors? (In the code on the left they have the same length.)

```
n = length(x);
p = 0;
for k = 1:n
    p = p + x(k)*y(k)
end

n = length(x);
m = length(y);
a = zeros(m,n);
for j = 1:m
    for k = 1:n
        a(j,k) = y(j)*x(k);
    end
end
```

Write two MATLAB commands that accomplish the same result as these two fragments.

2. Here is a table of three data points.

x	1	2	3
y	0	0	1

- Set up the Vandermonde system of equations for the power form $a_1 + a_2x + a_3x^2$ of the quadratic interpolating polynomial for these data.
- Find the coefficients a_1 , a_2 , and a_3 by solving the Vandermonde system of part(a).
- Find the coefficients of Newton's form of the interpolating polynomial.

[Exam continues on Reverse]

3. Suppose $y = Ax^2 + Bx + C$ is a quadratic function. Show by an algebraic calculation that the centered difference formula

$$C_h(a) = \frac{y(a+h) - y(a-h)}{2h}$$

delivers the exact derivative $y'(a)$ no matter what $h \neq 0$ is used.

Refer to the analysis of the centered difference formula, and explain why it delivers the exact derivative in this case.

4. The code fragment below estimates the probability P that a quadratic equation $Ax^2 + Bx + C$ with random coefficients A , B and C uniformly distributed in $[0, 1]$ has complex (that is, non-real) roots.

```
A = rand(1000,1); B = rand(1000,1); C = rand(1000,1);
NumCplx = 0
for k = 1:1000
    if (B(k)^2-4*A(k)*C(k)<0), NumCplx = NumCplx+1; end
end
P = NumCplx/1000;
```

Explain how the statement in the `for`-loop counts the equations that have complex roots.

That pedestrian `for`-loop can be vectorized and thus speeded up tremendously. How?

5. Refer to the script `Zoom` listed on page 44 that produces the plots in Figure 1.16 of page 45.

- (a) If x is close to 1, what is the largest term in the expansion of $p(x) = (x-1)^6$ given at the foot of page 43? When this term is evaluated in floating point arithmetic, how big do you expect the roundoff error to be in the computed result?
- (b) How well does your answer to part (a) account for the vertical scale of the rightmost subplot in the bottom row of Figure 1.16? (You may want to mention the roundoff errors in the remaining computed terms of $p(x)$ in your explanation.)