Math 273 Final Exam Solutions

1. On the jth trip through the loop, one division is performed in the first statement, and $j-1$ multiplications and $j-1$ subtractions in the second statement (a saxpy with vectors of length $j-1$). The total is

$$\sum_{j=2}^{n}(2j-1) = n^2 - 1.$$

Adding in the one division in the final statement gives a total of n^2 flops. This is similar to the analysis of forward substitution two pages earlier.

2. Set up A and b with

```
>> A = [x.^2 x ones(size(x))]; % NB: a(1) goes with x^2 !
>> b = y;
```

To compute `a`, MATLAB uses the QR algorithm to factor A into $Q \cdot R$, transforms the right side with Q^T, and finishes with back substitution.

3. The quantity `s` is the **k** component of the cross product of the Sun-Mercury vector with the Sun-Earth vector, divided by the product of their norms, so it is the sine of the angle between them.

At a time of conjunction, the sine of that angle is zero, so the text's assertion that the function is then zero is correct.

4. We learned in §1.5 that the optimum stepsize for numerical differentiation is $h = \sqrt{\rho/M}$, where ρ is a bound for the absolute error in evaluating f and M is a bound for $|f''|$ near x_c.

It's reasonable (page 46) to take $\rho = \texttt{eps} \cdot |f(x_c)|$, and prudent to allow $M = 100$ ($M = 1$ and $M = |f(x_c)|$ were also accepted). Finally, the increment size can be taken to be either absolute, or relative to the size of x_c, giving

$$\delta_c = \sqrt{\texttt{eps} \cdot |f(x_c)|/M} \qquad \text{or} \qquad \delta_c = \sqrt{\texttt{eps} \cdot |f(x_c)|/M} \cdot |x_c|.$$

5. Code for the first-order system and its Jacobian is as follows, assuming the obvious assignment of constants.

```
% ---------------------------------------------------------
function dydt = f(t,y)
dydt = [y(2); -1/(L*C)*y(1)-(R/L)*y(2)+(E0/L)*cos(w*t)];
% ---------------------------------------------------------
function dfdy = jacobian(t,y)
dfdy = [0 1; -1/(L*C) -(R/L)];
% ---------------------------------------------------------
```