1. First subtract $\frac{1}{2} \times$ row 1 from row 2 and $\frac{1}{3} \times$ row 1 from row 3 to obtain

$$
A = \begin{bmatrix}
60 & 30 & 20 \\
0 & 5 & 5 \\
0 & 5 & 5 \frac{1}{3}
\end{bmatrix}
$$

Then subtract row 2 from row 3 to make $A$ upper triangular; collecting the multipliers below the diagonal gives $L$:

$$
L = \begin{bmatrix}
1 & 0 & 0 \\
\frac{1}{3} & 1 & 0 \\
\frac{1}{3} & 1 & 1
\end{bmatrix}
$$

$$
U = \begin{bmatrix}
60 & 30 & 20 \\
0 & 5 & 5 \\
0 & 0 & \frac{1}{3}
\end{bmatrix}
$$

To solve $Ax = b = [0; 0; 1]$, first do forward substitution to solve $Ly = b$. The solution is obviously $y_1 = y_2 = 0$, $y_3 = 1$. Performing back substitution to solve $Ux = y$ gives

$$
\frac{1}{3}x_3 = 1 \implies x_3 = 3 \\
5x_2 + 5x_3 = 0 \implies x_2 = -3 \\
60x_1 + 30x_2 + 20x_3 = 0 \implies x_1 = \frac{1}{2}
$$

2. For $M$ we have the row and column sums

$$
0.1481 + 0.0740 = 0.2221 \\
+0.4445 + 0.2221 = 0.6666 \\
0.5926\ 0.2961
$$

The corresponding quantities for $M^{-1}$ are $10^8 \times$

$$
0.2221 + 0.0740 = 0.2961 \\
+0.4445 + 0.1481 = 0.5926 \\
0.6666\ 0.2221
$$
The infinity-norm is the maximum row sum:

\[ \|M\|_{\infty} = 0.6666 \quad \|M^{-1}\|_{\infty} = 0.5926 \times 10^8 \]

so \( \kappa_{\infty}(M) = 39502716 \).

The 1-norm is the maximum column sum:

\[ \|M\|_1 = 0.5926 \quad \|M^{-1}\|_1 = 0.6666 \times 10^8 \]

so \( \kappa_1(M) = 39502716 \) too.

3. MatVecRO computes the matrix-vector product \( y = Ax \) by a row-oriented algorithm using dot products. (The comments are correct about the result, but not the method.) For each dot product it computes \( n \) products of pairs of scalars \( (n \text{ flops}) \) and adds them \( (n \text{ flops more}) \) for a total of \( 2n \) flops. There are \( n \) dot products, making for \( 2n^2 \) flops.

MatVecCO computes the same product by a column-oriented algorithm using saxpys. (It is correctly described by the comments.) Each time through the loop a column of \( A \) is multiplied by a component of \( x \) \( (n \text{ flops}) \) and the result is added to \( y \) \( (n \text{ flops}) \). The loop is repeated \( n \) times for a total of \( n(n + n) = 2n^2 \) flops.

4. The matrix \( A \) and vector \( b \) are

\[
A = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \quad b = \begin{bmatrix} -5 \\ 5 \end{bmatrix}
\]

A rotation matrix \( G \) that makes \( A \) upper triangular is found by making the second component of \( GA \) zero \( (s = \sin \theta, \ c = \cos \theta) \):

\[
G \cdot A = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -3c + 4s \\ 3s + 4c \end{bmatrix}
\]

From \( 3 \sin \theta + 4 \cos \theta = 0 \) follows \( \cot \theta = -\frac{3}{4} \); thus \( \sin \theta = 1/\sqrt{1 + \cot^2 \theta} = 4/5 \) and \( \cos \theta = \sin \theta \cot \theta = -\frac{3}{5} \). Then \( A = QR \) with \( Q = G^T \) and \( R = [5, 0]^T \).

To solve the least squares problem, multiply the residual \( Ax - b \) by \( G \), obtaining

\[
GAx - Gb = Rx - Gb = \begin{bmatrix} 5 \\ 0 \end{bmatrix} x - \begin{bmatrix} 7 \\ 1 \end{bmatrix}
\]

The least squares solution is \( x^* = \frac{7}{5} \); the minimum residual is \( Ax^* - b = [4/5, 3/5] \), and its norm is 1.