

Math 273 Midterm II

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

1. Find unit lower triangular L and upper triangular U so that $A = LU$ for

$$A = \begin{bmatrix} 60 & 30 & 20 \\ 30 & 20 & 15 \\ 20 & 15 & 12 \end{bmatrix}$$

and use L and U to solve $Ax = [0; 0; 1]$.

2. Consider the matrix M and its inverse

$$M = \begin{bmatrix} 0.1481 & 0.0740 \\ 0.4445 & 0.2221 \end{bmatrix}, \quad M^{-1} = 10^8 \cdot \begin{bmatrix} 0.1481 & -0.0740 \\ -0.4445 & 0.2221 \end{bmatrix}.$$

Calculate the condition number $\kappa(M)$ with respect to any convenient norm.

Exam continues on Reverse

3. One of these M-files seems to have been produced by modifying the other, but the author forgot to change the comments.

```

function y = MatVecR0(A,x)      function y = MatVecC0(A,x)
% y = MatVecR0(A,x)            % y = MatVecC0(A,x)
% Computes the matrix-vector   % This computes the matrix-vector
% product y = A*x (via saxpys) % product y = A*x (via saxpys)
% where A is an m-by-n matrix % where A is an m-by-n matrix
% and x is a columnn-vector.  % and x is a columnn-vector.

[m,n] = size(A);               [m,n] = size(A);
y = zeros(m,1);                y = zeros(m,1);
for i=1:m                       for j=1:n
    y(i) = A(i,:)*x;           y = y + A(:,j)*x(j);
end                               end

```

- What does each function actually do, and how does it do it? [Do the comments correctly describe one of the codes?]
- Count the floating point operations required by each function to do its job in the case $m = n$.

4. To find the x that minimizes the sum of squares of residuals in the (overdetermined, inconsistent) equations

$$\begin{aligned} -3x &= -5 \\ 4x &= 5 \end{aligned}$$

- Determine the matrix A and vector b to express the system in the form $\min_x \|Ax - b\|$.
- Find an orthogonal matrix Q and upper triangular matrix R so that $A = QR$.
- Use the factorization to find x and the minimum residual.

Exam begins on Obverse