

## Math 273 Midterm I

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

1. The MATLAB fragment below approximates the *complete elliptic integral*  $F(k, \frac{\pi}{2})$  for  $k = .05, .10, .15, \dots, .50$  by computing partial sums of its Taylor series

$$F(k, \frac{\pi}{2}) = \frac{\pi}{2} \left[ 1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 k^4 + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 k^6 + \dots \right],$$

convergent for  $|k| < 1$ :

```
k = linspace(.05,.5,10)'; nterms = 50;
for i=1:10
    s = 1;
    for n = 1:nterms
        s = s + (prod(1:2:2*n-1)/prod(2:2:2*n))^2*k(i)^(2*n)
    end
    F(i) = pi/2*s;
end
```

Identify three MATLAB programming techniques that can make this fragment more efficient, and revise it accordingly.

2. Find the Newton form of the cubic polynomial that interpolates the data

$x$	-1	0	1	2
$y$	12	3	0	-9

**Exam continues on Reverse**

**3.** On p. 79 Van Loan shows how to evaluate a polynomial in the form of a sum of powers directly, then says

A more efficient algorithm is based on a nested organization of the polynomial, which we illustrate for the case  $n = 4$ :

$$p_3(x) = a_1 + a_2x + a_3x^2 + a_4x^3 = ((a_4x + a_3)x + a_2)x + a_1.$$

How many floating point operations does the evaluation of each form of this polynomial require?

**4.** What does the following MATLAB fragment do (assuming that a function M-file `function y=f1(x)` is available)?

```
x = linspace(1,2,101)';  
h = 1/100;  
for k = 1,100  
    df1(k) = (f1(x(k+1))-f1(x(k)))/h;  
end  
plot(x(1:100), df1)
```

- (a) Revise this fragment to achieve the same result more efficiently.
- (b) Replace the algorithm used in this fragment by one that is more accurate provided  $|f_1''|$  and  $|f_1'''|$  are of moderate size (say, both are smaller than 10).