Math 273 Midterm I

Carry out the solution of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular answers will receive no credit.

1. The MATLAB fragment below approximates the complete elliptic integral $F(k, \frac{\pi}{2})$ for $k = .05, .10, .15, \ldots, .50$ by computing partial sums of its Taylor series

$$F(k, \frac{\pi}{2}) = \frac{\pi}{2} \left[ 1 + \left( \frac{1}{2} \right)^2 k^2 + \left( \frac{1 \cdot 3}{2 \cdot 4} \right)^2 k^4 + \left( \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 k^6 + \ldots \right],$$

convergent for $|k| < 1$:

```matlab
k = linspace(.05,.5,10)'; nterms = 50;
for i=1:10
    s = 1;
    for n = 1:nterms
        s = s + (prod(1:2:2*n-1)/prod(2:2:2*n))^2*k(i)^(2*n)
    end
    F(i) = pi/2*s;
end
```

Identify three MATLAB programming techniques that can make this fragment more efficient, and revise it accordingly.

2. Find the Newton form of the cubic polynomial that interpolates the data

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>3</td>
<td>0</td>
<td>-9</td>
</tr>
</tbody>
</table>

Exam continues on Reverse
3. On p. 79 Van Loan shows how to evaluate a polynomial in the form of a sum of powers directly, then says

A more efficient algorithm is based on a nested organization of the polynomial, which we illustrate for the case $n = 4$:

$$p_3(x) = a_1 + a_2x + a_3x^2 + a_4x^3 = ((a_4x + a_3)x + a_2)x + a_1.$$  

How many floating point operations does the evaluation of each form of this polynomial require?

4. What does the following MATLAB fragment do (assuming that a function M-file function $y=f1(x)$ is available)?

```matlab
x = linspace(1,2,101)';
h = 1/100;
for k = 1:100
    df1(k) = (f1(x(k+1))-f1(x(k)))/h;
end
plot(x(1:100), df1)
```

(a) Revise this fragment to achieve the same result more efficiently.

(b) Replace the algorithm used in this fragment by one that is more accurate provided $|f_1''|$ and $|f_1'''|$ are of moderate size (say, both are smaller than 10).