

Math 267 §B2 Midterm NAME:_____

Carry out the *solution* of each problem: show steps of any required calculations; state reasons that justify any conclusions. Mere oracular *answers* will receive no credit.

1. For the initial value problem $y' = 1 + y^2$, $y(0) = a$:

(a) Find the solution.

(b) For what values of a does the solution exist for all t ?

2. A ball with mass 0.25 kg is thrown upward with initial velocity $v_0 = 20$ m/sec. Assume that the force due to air resistance has magnitude $|v|/40$ and take the acceleration of gravity $g = 9.8$ m/sec/sec.

(a) Find an expression for the velocity of the ball at any time t .

(b) Find maximum height that the ball attains, and compare to the height $\frac{v_0^2}{2g}$ that would be reached if there were no air resistance.

3. Find the *form* of a particular solution of each differential equation, by the method of undetermined coefficients. (*Do not solve for the coefficients.*) Include no unnecessary terms.

(a) $y'' + 4y = \sin 2t + te^{-2t}$

(b) $y'' + 16y = 2t \cos 4t + (t^2 - 4t)e^{-4t}$

4. One solution of $t^2y'' - t(t+2)y' + (t+2)y = 0$ is $y_1(t) = t$. Find a second, linearly independent solution.

5. A 4 kg mass is attached to a spring with spring constant 12 N/m; its motion is subject to a friction force of $1/2$ N sec/m times its velocity. Find the steady state response when the mass is driven by a force of $6 \cos 2t$ N. Express the motion in the form $A \cos(\omega t - \delta)$.