

3. (25 points) Recall that \mathbf{R}^+ denotes the set of positive real numbers. For $x, y \in \mathbf{R}$ we defined $x < y$ to mean $y - x \in \mathbf{R}^+$. Use this definition to prove: for $x, y, z \in \mathbf{R}$, if $x < y$ and $y < z$ then $x < z$. [HINT: Recall that \mathbf{R}^+ is closed under addition.]

4. (25 points) Use induction to prove: For all $n \in \mathbf{N}$,

$$\sum_{k=1}^n k(k+1) = n(n+1)(n+2)/3.$$