Math 201 1st Midterm Exam Name: _____

For full credit show the complete solution of each problem including steps of calculations. Support answers by citing known facts, definitions and theorems. No credit for mere answers unsupported by calculations or reasons.

1. With $\mathbf{Z}$ as the universal set, let $A = \{1, 2, 3\}$ and $B = \{3, 4\}$.
   (a) (10 points) Find $A \cup B$, $A \cap B$ and $B - A$.

   (b) (10 points) Describe the set $\bar{A}$ as $\{n \in \mathbf{Z} : p(n)\}$ using an open sentence $p(n)$ with domain $\mathbf{Z}$.

   (c) (10 points) Determine the set $A \times B$.

2. Let P and Q be statements.
   (a) (10 points) Is $(P \wedge Q) \Rightarrow P$ a tautology?

   (b) (10 points) Is the *converse* of the statement in part(a) a tautology?

   (c) (5 points) Is the *contrapositive* of the statement in part(a) a tautology?

3. (a) (10 points) Write the statement

$$\exists x, y \in \mathbf{Z},\ x^2 - y^2 = 71 \tag{1}$$

as an English sentence.

(b) (5 points) Write the negation of the statement (1) in symbolic form.

(c) (10 points) Write the negation of the statement (1) as an English sentence using the phrase "for all".

4. (20 points) Let $a, b \in \mathbf{Z}$. Prove without citing Theorem 3.16: If $a + b$ is odd then a and b have opposite parity.