MATH 201: LIPSCHITZ FUNCTIONS ARE INTEGRABLE
DUE FRIDAY, 30 NOVEMBER 2007

Prove:

Theorem 141.1. *If f satisfies a Lipschitz condition on $[a, b]$ then f is integrable on $[a, b]$.*

Plan for proof: Use a uniform partition P with intervals of constant width $x_i - x_{i-1} = h = (b-a)/n$ and a strategic choice of $n \in \mathbb{N}$. Apply Lemma 133.1(b) in every subinterval $[x_{i-1}, x_i]$ to get

$$\begin{aligned}
U(P,f) - L(P,f) &= \sum_{i=1}^{n} (M_i - m_i)(x_i - x_{i-1}) && \text{(Why?)} \\
&= h \sum_{i=1}^{n} (M_i - m_i) && \text{(Why?)} \\
&\le h \sum_{i=1}^{n} L \cdot h && \text{(Why?)} \\
&= nh^2 L && \text{(Why?)} \\
&< \varepsilon
\end{aligned}$$

if n was chosen large enough to begin with.