

# Mathematics 165T Problem Set 1

Week of January 15–19, 2001

All angles are expressed in radians.

The unit vector that makes an angle  $\theta$  with the positive  $x$ -axis is

$$\mathbf{u}(\theta) = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}.$$

In the following problems, angles  $\alpha$  and  $\beta$  satisfy  $0 \leq \beta \leq \alpha \leq 2\pi$ .

1. Show that

$$\|\mathbf{u}(\alpha) - \mathbf{u}(\beta)\| = 2 \sin \frac{1}{2}(\alpha - \beta).$$

2. Show that the vector  $\mathbf{u}(\alpha) - \mathbf{u}(\beta)$  makes an angle  $\varphi = \frac{\pi}{2} + \frac{1}{2}(\alpha + \beta)$  with the positive  $x$ -axis.

3. Using the polar form, with  $\varphi$  from Problem 2,

$$\mathbf{u}(\alpha) - \mathbf{u}(\beta) = \|\mathbf{u}(\alpha) - \mathbf{u}(\beta)\| \mathbf{u}(\varphi)$$

deduce the trigonometric identities

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha - \beta) \sin \frac{1}{2}(\alpha + \beta)$$

$$\sin \alpha - \sin \beta = +2 \sin \frac{1}{2}(\alpha - \beta) \cos \frac{1}{2}(\alpha + \beta)$$